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FORCES ON A HORIZONTAL CYLINDER
DUE TO NON-LINEAR WAVES

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THESIS

FORCES ON A HORIZONTAL CYLINDER DUE TO NON-LINEAR WAVES

by

Fred Herman Gehrman Jr.

Thesis Advisor: C.J. Garrison

December 1972

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Forces on a Horizontal Cylinder Due to Non-Linear Waves

by

Fred Herman Gehrman, Jr.
Lieutenant, United States Navy
B. S., The Creighton University, 1966

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
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ABSTRACT

A horizontal cylinder, located near the floor of a two dimensional wave channel was subjected to a train of non-linear gravity waves. The horizontal and vertical forces were measured and presented in dimensionless form. Experimental values of horizontal and vertical force coefficients are presented as functions of dimensionless wave height and dimensionless wave period. The dimensionless force coefficients predicted by a modified Morrison equation and a Froude-Krilov force are compared to experimental data. Fluid particle velocity and acceleration values were calculated from Stokes fifth-order gravity wave theory. Experimental dimensionless wave periods from 10 to 200 were investigated.

The horizontal force coefficients were found to vary linearly with wave height for dimensionless period values from 60 to 120. The vertical force coefficients were found to be inertia dominated at low dimensionless wave heights and dominated by a lift force at higher wave heights. The theory predicted experimental values of force coefficients with good accuracy, especially at greater water depths.

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LIST OF SYMBOLS

SYMBOL	DEFINITION	UNITS
a	cylinder radius	ft.
a_x	horizontal acceleration	ft./sec.^{2}
$\tilde{a}_x$	dimensionless horizontal acceleration	
a_y	vertical acceleration	ft./sec.^{2}
$\tilde{a}_y$	dimensionless vertical acceleration	
C	wave celerity	ft./sec.
$\tilde{C}$	dimensionless wave celerity	
C_D	drag coefficient	
C_L	lift coefficient	
C_M	added mass coefficient	
$\tilde{d}$	depth to wave length ratio	
F_x	horizontal force component	lbf.
$\tilde{F}_x$	dimensionless horizontal force component	
F_y	vertical force component	lbf.
$\tilde{F}_y$	dimensionless vertical force component	
F_i	vertical inertial force	lbf.
$\tilde{F}_i$	dimensionless vertical inertial force	
F_L	vertical lift force	lbf.
$\tilde{F}_L$	dimensionless lift force	
g	gravitational constant	ft./sec.^{2}

SYMBOL	DEFINITION	UNITS
gT^2/h	dimensionless period parameter	
h	water depth	ft.
$\tilde{h}$	dimensionless water depth	
H	peak to trough wave height	ft.
$\tilde{H}$	wave height to diameter ratio, dimensionless wave height	
$\tilde{H}d$	wave height to depth ratio	
L	wave length	ft.
ℓ	cylinder length	ft.
P	pressure	lbf./ft.^{2}
$\tilde{P}$	dimensionless pressure	
S	position measured from bottom	ft.
u	horizontal velocity	ft./sec.
$\tilde{u}$	dimensionless horizontal velocity	
v	vertical velocity	ft./sec.
$\tilde{v}$	dimensionless vertical velocity	
y	distance from mean water line	ft.
$\tilde{y}$	dimensionless distance from mean water line	
ρ	fluid density	lbm./ft.^{3}
ϕ	velocity potential	ft.^{2}/sec.
$\tilde{\phi}$	dimensionless velocity potential	

SYMBOL	DEFINITION	UNITS
λ	arbitrary constant in Stokes Wave Theory	
η	distance from mean water line to free surface	ft.
θ	phase angle	degrees

ACKNOWLEDGEMENTS

The author wishes to express his appreciation, first of all to his advisor, Professor C. J. Garrison, for his enlightenment, encouragement and patience throughout this study. In addition, special thanks is also given to Mr. Tom Christian, who designed the wave height probe without which this study could not have been completed. Finally and most deeply, to Claire, Chris and Karen, for their patience, understanding and long-suffering during this undertaking, the author expresses his love.

I. INTRODUCTION

Exploration for and use of ocean resources has received increasing attention in the scientific community in recent years. A considerable effort has resulted in the development of technology in areas such as off-shore oil exploration, recovery of petroleum and deployment of underwater habitats. Oil production often involved the use of very large submerged oil storage tanks and associated piping systems. In some locations, oil is transferred long distances in submerged piping systems, often leading through the surf zone to facilities ashore. Also, waste disposal systems usually involve deployment of large diameter outfall pipes which are laid through the surf zone. Proposals for large scale mining and food gathering activities indicate the trend toward design and construction of large submerged structures.

This activity has generated considerable interest in the interaction of gravity waves with submerged objects and particularly in the forces induced. Most previous investigations [1,2,3] in this area center on the use of a so-called Morrison Equation [3], which involves both a drag and inertial component of force. Unfortunately, the drag and added mass coefficients are found to be neither constant nor simply shape - or Reynolds number - dependent in the unsteady flow that occurs in the interaction of waves with fixed bodies. In addition, where the amplitude of wave motion is large compared to the body dimensions, flow separation occurs,

and the coefficients vary as the flow in a given direction develops. Finally, for small depth to characteristic dimension ratios, the proximity of the free surface may influence the value of the coefficients, even if no flow separation occurs.

In spite of these difficulties, the Morrison Equation will, with certain restrictions, provide a complete description of the horizontal forces acting on a large horizontal cylinder in contact with a plane boundary. If the cylinder is large in comparison to the amplitude of the fluid motion, separation does not occur, and the resulting wave-induced oscillatory flow may be considered an unseparated potential flow about the cylinder. This has been verified by Kuelegan and Carpenter [2] and by Sarpkaya and Garrison [4]. For such cases, the drag component of force is negligible, while the added mass coefficient is a constant.

Application of the Morrison Equation to determine the vertical components of force for a bottom-mounted cylinder yields a vertical force component of zero, an obviously invalid result. It is recognized that the vertical force should be composed of two components; one associated with the inertia effects and a second which accounts for the lift force caused by the increased velocity and hence decreased the pressure over the top of the cylinder.

The inertial component of vertical force can be approximated by what is commonly known as a Froude-Krilov force. This is the force caused by the pressure distribution around

the surface of the cylinder existing in the wave if the cylinder was not present. This is an approximation which does not account for the presence of the body and is therefore considered to be an underestimate of the force.

The lift component of vertical force can be approximated by use of the unseparated, potential flow model used in the analysis of the horizontal forces, i.e., that of a uniform flow past a cylinder in contact with a rigid boundary. The lift coefficient for this case has been determined by Dalton and Helfenstein [5].

A study carried out by Johnson [6] for horizontal forces acting on a bottom-mounted, horizontal cylinder in long waves, assumed that at long wave lengths, the horizontal forces depended on wave height and water depth only. This assumption is, however, of doubtful validity in the range of Johnson's test.

Shiller [7] measured wave forces on a submerged horizontal cylinder due to small amplitude waves. Over the range of wave heights considered, the magnitude of the horizontal force was found to vary linearly with wave height. The magnitude of the vertical force was found to increase in proportion to the wave height squared.

Perkinson [8] extended Shiller's investigation to include larger wave heights and periods. His study indicates that horizontal forces increased linearly with wave heights and becomes independent of wave period for large wave periods. The vertical forces were found to contain two regimes: the

lower wave lengths where lift force predominates; and the higher wave lengths where inertial force is dominant.

In order to evaluate any forces using the above analyses, it is necessary to first evaluate fluid velocities, accelerations and pressure by use of some wave theory. Fairly simple expressions may be derived for the forces by use of Airy wave theory. However, for finite amplitude waves in finite depth water, a non-linear wave theory is considered to be necessary. Stokes fifth-order wave theory has been chosen for this study.

Stokes originally developed a second-order theory [9] for the case of a non-linear wave in water of finite depth. This method has been extended by Borgman and Chappelear [10] to third order. Skjelbreia and Hendrickson [11] have extended the solution to third and fifth order. Bretschneider [12] has presented a method for extension to any order.

The purpose of this study is to compare the analytical results of Stoke's fifth-order wave theory with experimental values at longer wave periods. The transition section of the wave channel used by Perkinson was extended approximately forty feet to provide a slope of 1:20. This provided the possibility of extending Perkinson's work to include longer wave periods. This study, then, is an extension of the work of Shiller [7] and Perkinson [8].

II. THEORETICAL ANALYSIS

A. PROBLEM DEFINITION

The problem under consideration is illustrated in Figure 1. A train of gravity waves is considered to progress in the positive x-direction in water of depth h, with rate of propagation C. The fluid particle velocity is expressed in terms of the horizontal and vertical components, u and v. It is of primary interest to determine the horizontal and vertical components of wave force acting on the horizontal cylinder which is in contact with the rigid bottom.

B. DIMENSIONAL ANALYSIS

An exact analytical solution to this problem is quite formidable and, therefore, an approximate solution is considered in this work. However, first it is instructive to carry out a dimensional analysis of all pertinent parameters involved.

In general, the maximum of the wave force per unit length acting on a submerged cylinder due to wave motion is known a priori to depend on the following parameters:

$$F_{max}/\ell = f_1(h,H,T,L,a,\rho,g,\mu) \qquad (1)$$

where

F = wave force
h = water depth
H = wave height
T = wave period
L = wave length

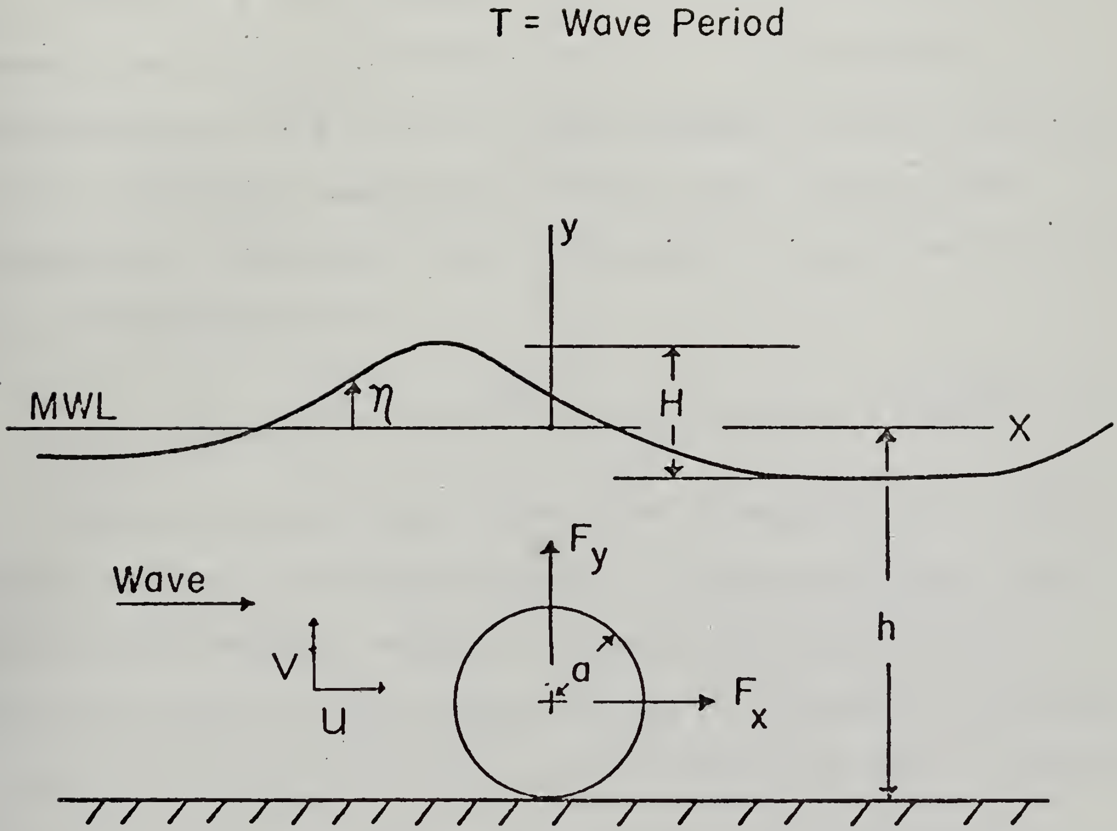


Fig. I. DEFINITION OF GEOMETRY

a = cylinder radius
ρ = fluid density
g = gravitational constant
μ = fluid viscosity
ℓ = cylinder length

However, a relationship exists between the parameters associated with the incident wave (i.e. h,H,T and L). Consequently, only three of these parameters (e.g. h,H and T) are needed to completely specify the incident wave. A dimensional analysis of the parameters in equation (1) yields the following groups:

$$F_{max}/\rho g a^2 \ell = f_2(gT^2/h,\ h/a,\ H/2a,\ \mu/\ell\sqrt{gha^2}) \quad (2)$$

The last term on the right side of equation (2) represents the ratio of Froude number to Reynolds number, indicating the ratio of viscous to inertial forces. It is believed that, if this number is small, it may be neglected from further consideration, and equation (2) may be rewritten

$$\tilde{F}_{max} = f_3(gT^2/h,\ \tilde{H},\ \tilde{h}) \quad (3)$$

where $\tilde{F}_{max} = F_{max}/\rho g a^2 \ell$
$\tilde{H} = H/2a$
$\tilde{h} = h/a$

The approximate analytical approach of this study seeks:

1. To develop expressions for the fluid particle motion in the gravity wave as functions of the incident wave parameters, and

2). To develop expressions for the forces on the submerged cylinder as functions of the fluid particle motion.

The assumptions upon which this analysis is based are:

1). Water will be considered inviscid and incompressible,

2). The cylinder radius will be considered small in comparison to the dimensions (h and L) of the incident wave.

C. WAVE THEORY

The motion of fluid particles in a gravity wave based on assumptions 1) and 2) above is specified by the following boundary value problem.

1). Governing Equation

Assumption 1) implies that Laplace's Equation governs the fluid motion. That is:

$$\nabla^2\phi = 0 \tag{4a}$$

where ϕ is the velocity potential and the two velocity components are given in terms of ϕ as

$$u = \frac{\partial\phi}{\partial x}\ , \quad v = \frac{\partial\phi}{\partial y} \tag{4b}$$

2). Boundary Conditions

The boundary condition on the bottom is that of a non-porous wall, i.e.,

$$\frac{\partial\phi}{\partial y} = 0 \quad \text{at } y = -h \tag{5}$$

Two boundary conditions are encountered at the water surface, one dynamic and one kinematic.

The dynamic boundary condition is obtained from Bernoulli's Equation by setting the pressure equal to zero. That is:

$$u^2 + v^2 + 2\frac{\partial \phi}{\partial t} = -2g(\Pi - y) \quad \text{at } y = \eta \qquad (6)$$

where Π is the total energy head, a constant.

Assuming the wave travels without change in form, it is then possible to choose a reference system moving in the positive x direction with wave celerity C. This makes the fluid motion steady with respect to the moving reference system. The Bernoulli Equation in such a reference system is:

$$(u - C)^2 + v^2 = 2g(\Pi - y) \quad \text{at } y = \eta \qquad (7)$$

The kinetic boundary condition imposes the condition that no fluid be transported across the free surface. Or, in other words, the free surface must be a streamline. This may be expressed mathematically in the form:

$$\frac{\partial y}{\partial x} = \frac{v}{u - C} \quad \text{at } y = \eta \qquad (8)$$

Equations (4a), (5), (7), and (8) completely specify the boundary value problem.

There are, historically, several approximate solutions to this problem. The solution to be considered further in this analysis is that commonly known as Stokes fifth-order wave theory.

D. STOKES FIFTH ORDER SOLUTION

Stokes proposed [9] a power series type of solution to the above problem and proved the validity of the second order solution based on this power series. Skjelbreia and Hendrickson [9] have extended this theory to fifth-order, yielding a solution to the boundary value problem of the form:

$$\phi = \frac{C}{\beta}\{A_1\cosh(\beta S)\sin\theta + A_2\cosh(2\beta S)\sin(2\theta) + A_3\cosh(3\beta S)\sin(3\theta) + A_4\cosh(4\beta S)\sin(4\theta) + A_5\cosh(5\beta S)\sin(5\theta)\} \qquad (9a)$$

where A_1 through A_5 are as given in Appendix A.

This equation may be written in dimensionless form as:

$$\tilde{\phi} = \tilde{C}\{A_1\cosh(\beta S)\sin\theta + A_2\cosh(2\beta S)\sin(2\theta)+\ldots\} \qquad (9b)$$

where: $\tilde{\phi} = \beta\phi/\sqrt{gh}$

$\tilde{C} = C/\sqrt{gh}$

Differentiation of equation (9a) with respect to time yields

$$\frac{\partial\phi}{\partial t} = C^2\{A\cosh(\beta S)\cos\theta + 2A_2\cosh(2\beta S)\cos(2\theta) + 3A_3\cosh(3\beta S)\cos(3\theta) + 4A_4\cosh(4\beta S)\cos(4\theta) + 5A_5\cosh(5\beta S)\cos(5\theta)\} \qquad (10a)$$

or, in dimensionless form

$$\frac{\partial\tilde{\phi}}{\partial t} = \tilde{C}^2\{A_1\cosh(\beta S)\cos(\theta) + 2A_2\cosh(2\beta S)\cos(2\theta) + \ldots\} \quad (10b)$$

where $\frac{\partial\tilde{\phi}}{\partial t} = \frac{\partial\phi}{\partial t}/\sqrt{gh}$

The series form of the wave profile is assumed to be:

$$y = \frac{1}{\beta}\{\lambda\cos(\theta) + B_2\cos(2\theta) + B_3\cos(3\theta) + B_4\cos(4\theta) + B_5\cos(5\theta)\} \quad (11a)$$

where B_2 through B_5 are as given in Appendix A and λ is an arbitrary constant.

Equation (11a) can be expressed in dimensionless form as

$$\tilde{y} = \frac{1}{\tilde{d}}\{\lambda\cos(\theta) + B_2\cos(2\theta) + \ldots\} \quad (11b)$$

where $\tilde{y} = y/d$

$\tilde{d} = 2\pi h/L$

The series form for Π is assumed to be:

$$\Pi = \frac{1}{\beta}\{\lambda^2 C_3 + \lambda 4 C_4\} \quad (12)$$

where C_3 and C_4 are as given in Appendix A.

The wave celerity expressed in series form is

$$C = \frac{y}{\beta}\, C_o^2\{1 + \lambda^2 C_1 + \lambda 4 C_2\} \quad (13a)$$

or, in dimensionless form

$$\tilde{C} = \frac{1}{\tilde{d}} C_o^2\{1 + \lambda^2 C_1 + \lambda^4 C_2\} \tag{13b}$$

where C_1 and C_2 are as given in Appendix A.

Using equations (4b) and (9b) we find the horizontal particle velocity to be:

$$\tilde{u} = (\frac{\partial\tilde{\phi}}{\partial x}) = \tilde{C}\{A_1\cosh(\beta S)\cos(\theta) + 2A_2\cosh(2\beta S)\cos(2\theta)$$

$$+ 3A_3\cosh(3\beta S)\cos(3\theta) + 4A_4\cosh(4\beta S)\cos(4\theta)$$

$$+ 5A_5\cosh(5\beta S)\cos(5\theta)\} \tag{14}$$

where $\tilde{u} = u/\sqrt{gh}$

Also, from equations (4b) and (9a):

$$\tilde{v} = (\frac{\partial\tilde{\phi}}{\partial y}) = \tilde{C}\{A_1\sinh(\beta S)\cos(\theta) + 2A_2\sinh(2\beta S)\cos(2\theta)$$

$$+ 3A_3\sinh\ (3\beta S)\cos(3\theta) + 4A_4\sinh(4\beta S)\cos(4\theta)$$

$$+ 5A_5\sinh(5\beta S)\cos(5\theta)\} \tag{15}$$

where $\tilde{v} = v/\sqrt{gh}$

Differentiation of equations (14) and (15) with respect to time yields the particle accelerations

$$\tilde{a}_x = (\frac{\partial\tilde{u}}{\partial t}) = \tilde{d}\tilde{C}^2\{A_1\sinh(\beta S)\sin(\theta) + 2A_2\sinh(2\beta S)\sin(2\theta)$$

$$+ 3A_3\sinh(3\beta S)\sin(3\theta) + 4A_4\sinh(4\beta S)\sin(4\theta)$$

$$+ 5A_5\sinh(5\beta S)\sin(5\theta) \tag{16}$$

where $\tilde{a}_x = a_x/g$

and

$$\tilde{a}_y = \tilde{d}\tilde{C}^2\{A_1\sinh(\beta S)\cos(\theta) + 2A_2\sinh(2\beta S)\cos(2\theta) + \ldots\} \tag{17}$$

where $\tilde{a}_y = a_y/g$

The potential function and it's derivatives, as well as the wave profile are now known in terms of the unknown quantities λ and $\tilde{d}$. Specifying equations for these parameters are found by developing equations for the peak-to-trough wave height and the wave period. That is, the dimensionless wave height is defined as:

$$\tilde{H}d = \tilde{y}_{\theta=0} - \tilde{y}_{\theta=\pi} \tag{18}$$

where $\tilde{H}d = H/h$

Substitution of equation (13b) into equation (18) yields:

$$\tilde{H}d = \frac{2}{\tilde{d}}\{\lambda + \lambda^3 B_{33} + \lambda^5(B_{35} + B_{55})\} \tag{19}$$

where B_{33}, B_{35}, and B_{55} are as given in Appendix A.

Now, the wave period and the wave celerity are related by the relationship:

$$C = L/T \tag{20}$$

So, equation (15a) becomes

$$(L/T)^2 = g\frac{C_o^2}{\beta}\{1 + \lambda^2 C_1 + \lambda^4 C_2\} \tag{21}$$

or,

$$h/gT^2 = \frac{\tilde{d}}{4\Pi^2} C_o^2 \{1 + \lambda^2 C_1 + \lambda^4 C_2\} \qquad (22)$$

The parameter gT^2/h is referred to herein as the period parameter.

Equations (19) and (22), if solved simultaneously, yield values for λ and $\tilde{d}$ in terms of $\tilde{H}d$ and gT^2/h, quantities which are incident wave parameters. Thus the potential function, particle velocities and accelerations are shown to be functions of the dimensionless parameters obtained by dimensional analysis and given by equation (3). (Note that $\tilde{H}d = 2H/h$)

The next step in the analysis is to express the forces acting on the cylinder as functions of the potential function, particle velocities and accelerations.

E. FORCES ON THE CYLINDER

The forces acting on the cylinder are described in terms of their horizontal and vertical components.

1). Horizontal Component

The horizontal component of force acting on the cylinder is expressed in the form of the so-called Morrison Equation as.

$$F_x = \frac{C_D}{2}(\rho 2a\ell u^2) + (1.0 + C_M)\pi a^2 \ell a_x \qquad (23)$$

where C_D = drag coefficient

C_M = added mass coefficient

For cases where fluid particle motion is small in comparison with the cylinder diameter, flow separation does not occur and the contribution of drag to the total force may be disregarded. In this case, equation (23) may be rewritten

$$\tilde{F}_x = (1.0 + C_M)\tilde{a}_x \tag{24}$$

where $\tilde{F}_x = F_x/\rho g a^2 \ell$.

The added mass coefficient for a circular cylinder in contact with a rigid wall was given in closed form by Garrison [13] as $C_M = 2.29$.

2). Vertical Force Component

As mentioned previously, application of the Morrison Equation to the vertical component of force yields a result which is clearly erroneous. The vertical force may be considered to arise from two sources: an inertial force and a lift force.

a). Lift Force

The lift force may be expressed as:

$$F_L = 1/2(2a\ell\rho)C_L u|u| \tag{25}$$

where C_L = lift coefficient.

or, in dimensionless form:

$$\tilde{F}_L = \tilde{h}C_L u|u| \tag{26}$$

where $\tilde{F}_L = F_L/\rho g a^2 \ell$.

Dalton and Helfstein indicate [7] that $C_L = 4.49$ for unseparated flow past a circular cylinder in contact with a rigid wall.

b). Inertial Force

The pressure distribution around the surface of the cylinder existing if the cylinder is not present results in a net vertical force (known as a Froude-Krilov force).

Consider the resultant differential force dF_i acting on a differential area of the cylinder dA as shown in Figure 2. The above equation, written in dimensionless form, is:

$$\tilde{F}_L = \int_0^{2\pi} \tilde{P}(a,\gamma)\sin\gamma d\gamma \tag{30}$$

The pressure is obtained from the Bernoulli Equation:

$$P = -\rho\{\frac{1}{2}(u^2 + v^2) + \frac{\partial\phi}{\partial t}\} \tag{31}$$

or, in dimensionless form:

$$\tilde{P} = -\tilde{h}[\frac{1}{2}(\tilde{u}^2 + \tilde{v}^2) + (\frac{\partial\tilde{\phi}}{\partial t})] \tag{32}$$

where $\tilde{P} = P/\rho ga$

Therefore

$$\tilde{F}_i = -\tilde{h} \int_0^{2\pi} [\frac{1}{2}(\tilde{u}^2 + \tilde{v}^2) + (\frac{\partial\tilde{\phi}}{\partial t})]\sin\gamma dY \tag{33}$$

And the total vertical force may be written:

$$\tilde{F}_y = \tilde{h}\{C_L\tilde{u}|\tilde{u}| - \int_0^{2\Pi} [\frac{1}{2}(\tilde{u}^2 + \tilde{v}^2) + (\frac{\partial\tilde{\phi}}{\partial t})]\sin\gamma d\gamma\} \tag{34}$$

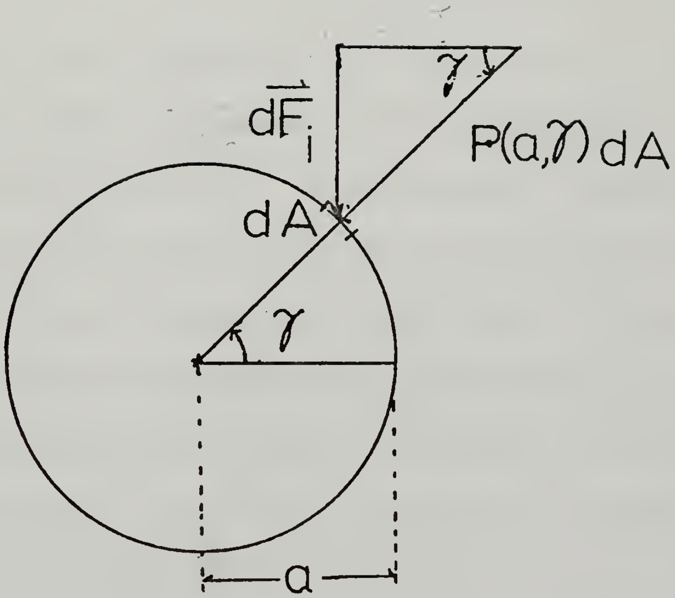


Fig. 2

INERTIAL FORCE GEOMETRY

F. DISCUSSION OF COMPUTER PROGRAM

The equations developed above were programmed for a digital computer as given in Appendix D. The basic inputs to the program are $\tilde{H}$, $\tilde{h}$ and gT^2/h. The program yields particle velocity, acceleration and computes the horizontal and vertical forces on the cylinder at various wave angles.

The STAKES5 subroutine calculates the coefficients for the Stokes fifth order equations. Simultaneous solution to equations (19) and (22) are obtained by assuming an initial value of λ and $\tilde{d}$ based on linear wave theory and iterating to the actual value using the Newton-Raphson technique.

The main program generates tables of values and in-line graphs of the wave profile, particle velocity vs. depth, vertical and horizontal force vs. phase angle.

III. DESCRIPTION OF APPARATUS AND EXPERIMENTAL PROCEDURE

A. WAVE CHANNEL

Wave forces on the cylinder were measured experimentally in a wave channel. The basic wave channel, as shown in Figure 3, was fifteen inches wide and consisted of two sections: a transition section and a shallow water section. The overall channel length was 87 feet.

Three-quarter inch exterior plywood sheets were used for the channel sides. Vertical rigidity was provided by bracing the plywood sheets with 2x4's at various intervals. The vertical members were placed four feet on centers in the shallow water section and from four feet to eighteen inches in the transition section. Additional rigidity was provided in the transition section by truss arrangements, as shown in Figure 4. Two 2x4's were attached to the top of the vertical studs and extended the length of the channel.

A double floor constructed of two plywood sheets with two half-inch separators was used. The sides of the channel were bolted together, through the spacing of the double bottom, with three-eights inch threaded rod.

All wooden sections exposed to water were water-proofed and painted with three coats of Morewear Vitri-Glaz 1320A. Dow Corning Sealant 780 was applied to all seams and joints.

A paddle type wave generator was used to generate waves in the channel. An aluminum plate was hinged at the tank

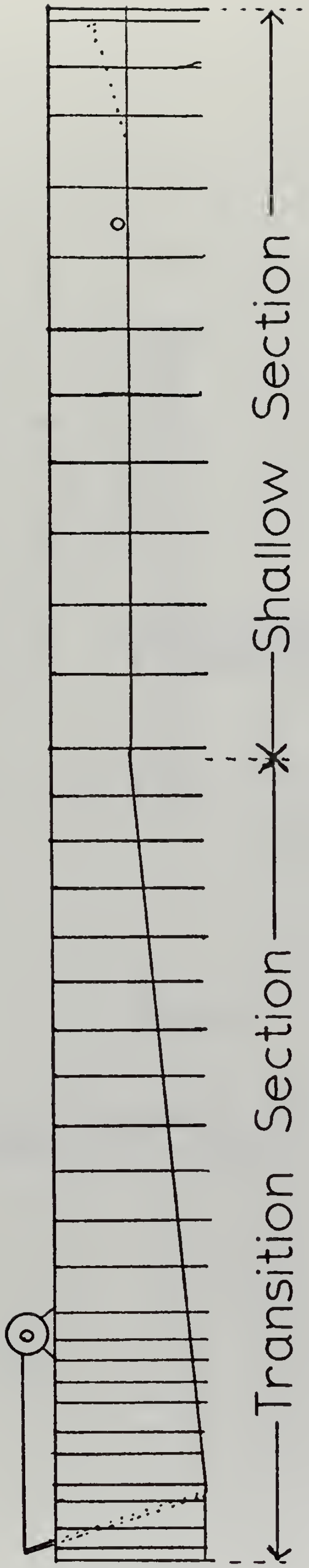


Fig.3 WAVE CHANNEL

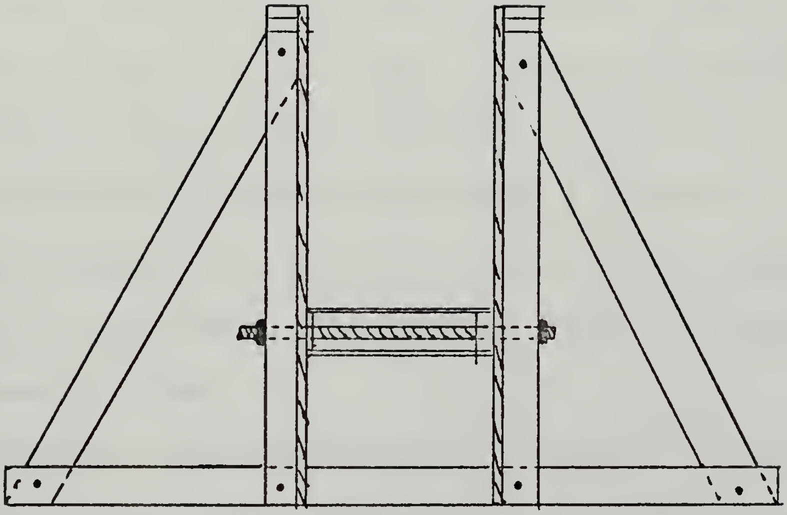


Fig.4

TRANSITION SECTION TRUSS ARRG'T.

floor and attached by a pin connection at the top to a driving rod. Wave motion behind the paddle was damped by use of baffle plates. A two horse power variable speed drive, with an output speed range of twenty to one hundred eighty revolutions per minute, was mounted on top of the wave channel and fitted with a six and three quarter inch radius face plate. Ways were attached to the plate to adjust the driving rod eccentricity between zero and eight inches.

The transition section contained a ramp with a slope of 1:20 which shoaled the generated wave with a change in depth from four to two feet. The overall length of the transition section was 42 feet.

The shallow water section had a depth of two feet and an overall length of 42 feet. The test module was located twenty four feet from the transition, allowing the waves to reach a fully developed state following the transition before reaching the test section.

A variable slope dissipater beach was located at the end of the test section in order to dissipate the wave motion. The beach was constructed of metal shavings held between two pieces of perforated stainless steel sheet metal, which were separated by two inch wooden spacers. A solid sheet of one-eighth inch aluminum was attached to the bottom of the beach by three and five eighths inch separators. One by one inch aluminum angles were attached to the exposed surface of the beach, parallel to the wave fronts. Maximum slope of the beach was 1:7.

B. TEST MODULE

The test module consisted of the circular cylinder with its own floor and walls, constructed as an integral system to provide easy removal of the model from the tank. The wave channel sides and floor were recessed to provide a smooth transition from channel to test module. The channel walls were made of plexiglass in the test module region to allow maximum visibility of the model. The test module is shown in Figure 5.

A four inch diameter plexiglass cylinder was supported between the walls of the test module by use of cantilever beams and adjusted to approximately one-sixteenth inch above the floor. A thin flexible plastic barrier was installed in this gap to prevent any water motion under the cylinder due to wave motion. The barrier was held in place by 'O' ring material pressed into 0.007 inch slots in the cylinder and test module floor.

The cantilever beams were used to support the cylinder, as shown in Figures 6 and 7. Bulkheads were fixed in the cylinder at approximately two inches from each end, and the fixed ends of the cantilever beams bolted on these bulkheads. The free ends of the two beams protruded slightly beyond the end of the cylinder and into the plexiglass test module walls where they were supported by small self-aligning ball bearings pressed into the plexiglass. Both beams were fitted with strain gauges and waterproofed using BLH Barrier 'C', as shown in Figure 8.

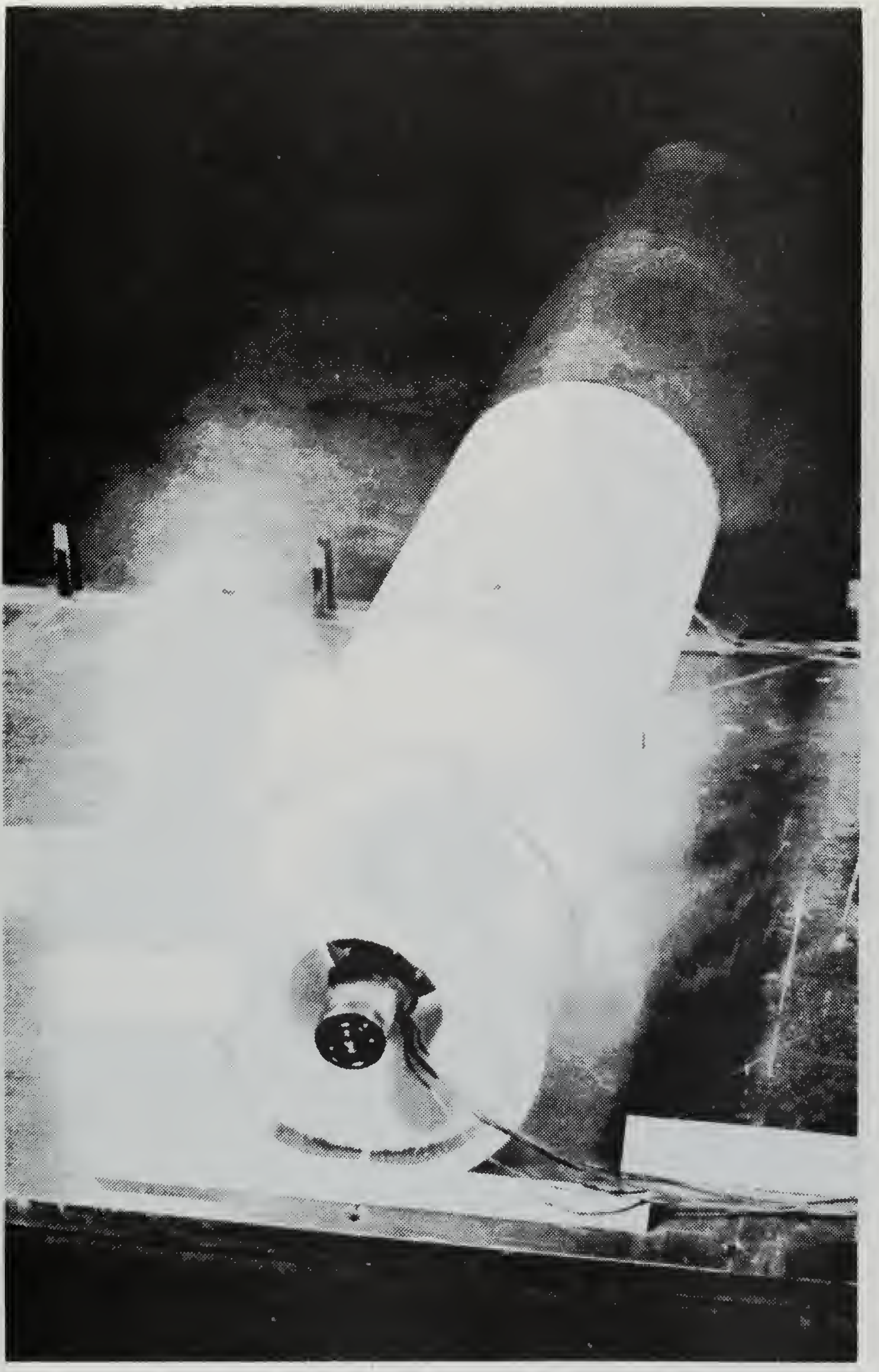


Fig. 5 TEST MODULE

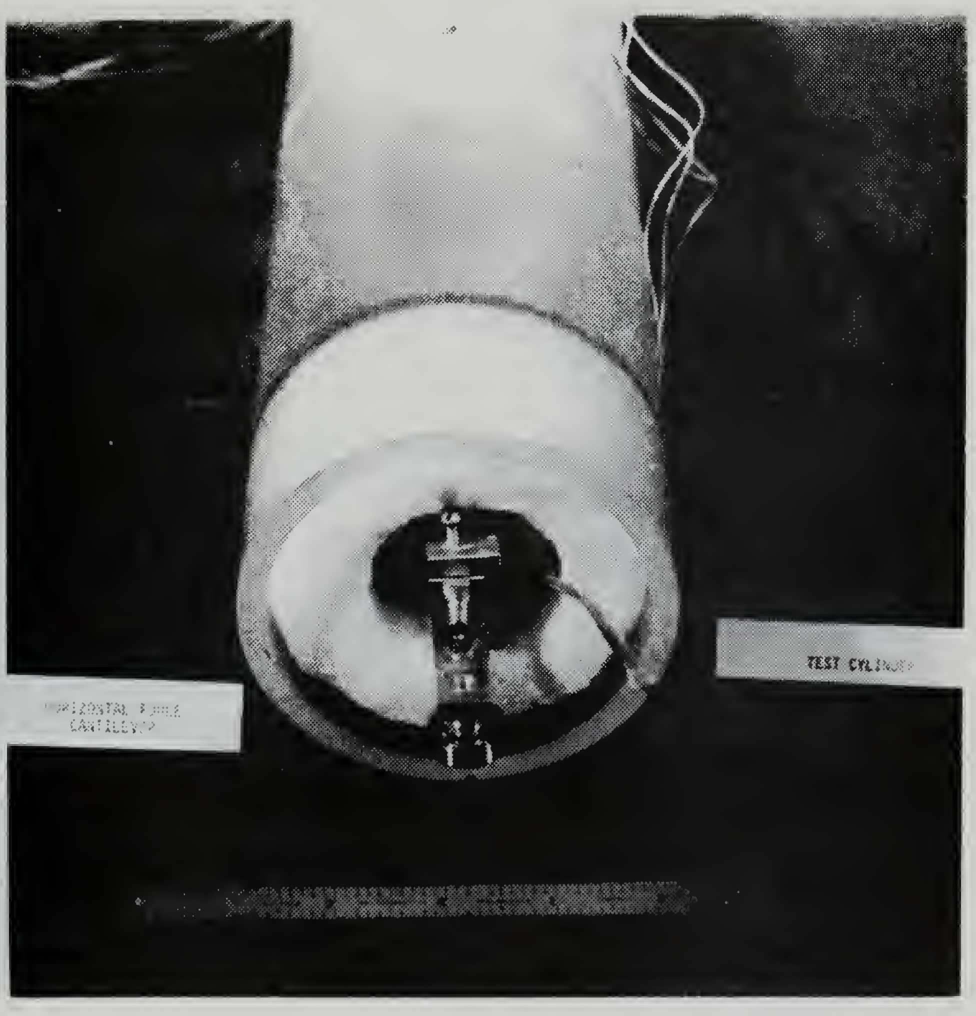


Fig. 6 HORIZONTAL FORCE CANTILEVER

[illegible]

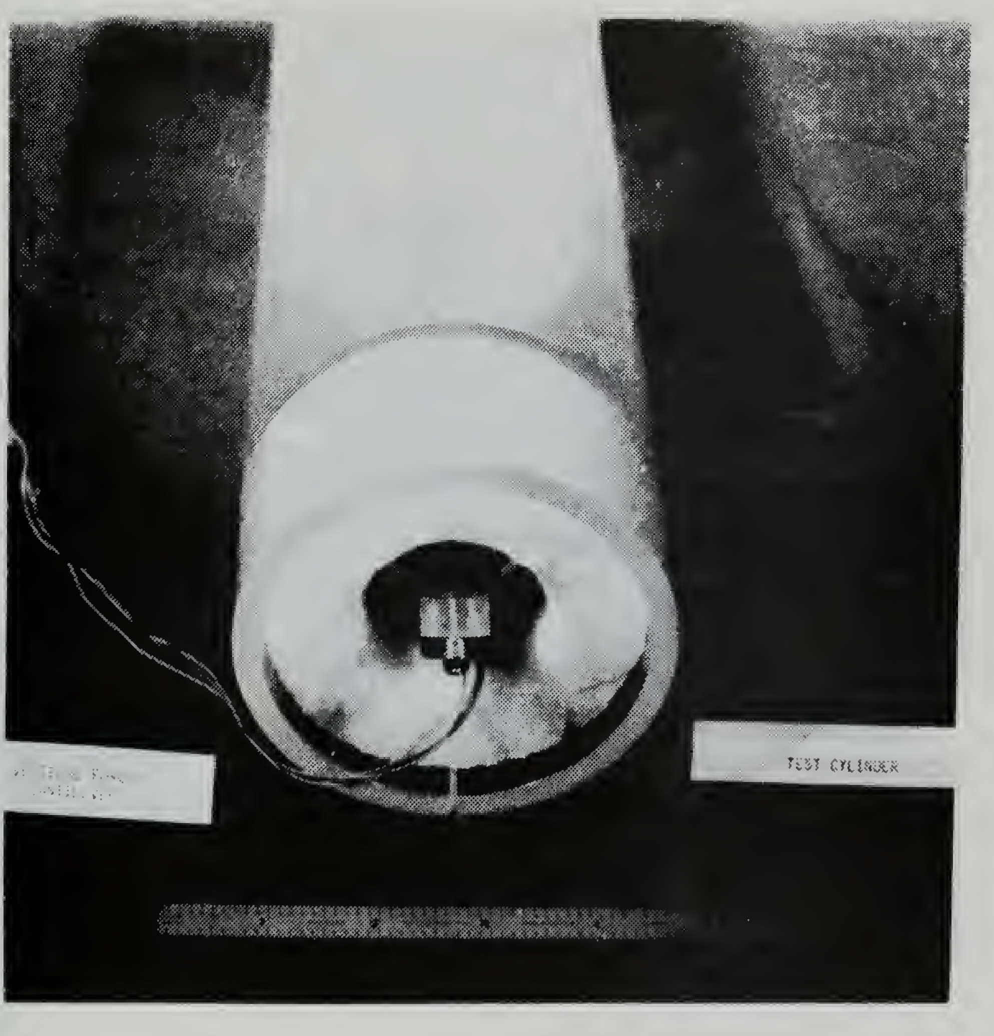


Fig. 7 VERTICAL FORCE CANTILEVER

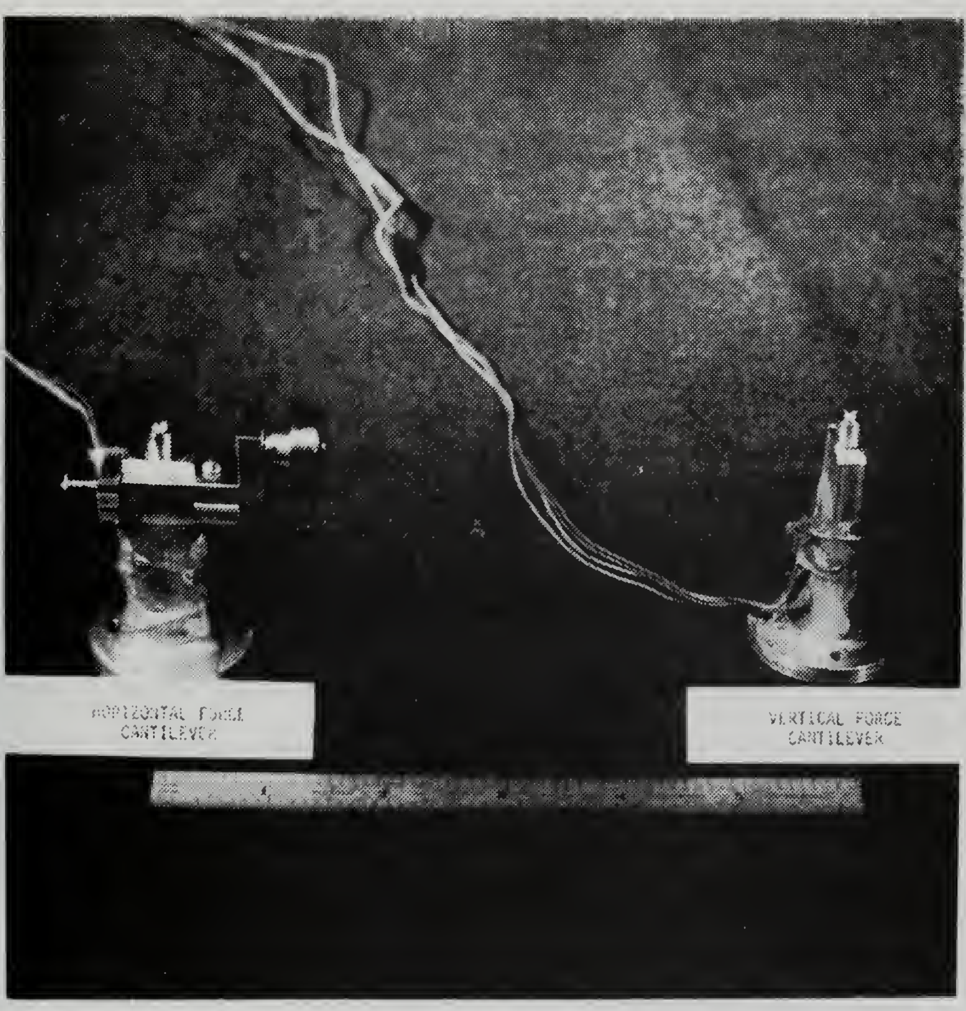


Fig. 8 FORCE CANTILEVER

One beam, positioned with its largest cross sectional dimension vertical, was used to measure horizontal force and the other, with its largest cross section positioned horizontally, was used to measure vertical force. Calibration tests indicated that cross coupling was negligible.

Dimensions of the beams were chosen to provide sufficient flexibility to allow measurement of forces using strain gauges and sufficient stiffness to provide a natural frequency that was large in comparison with the excitation frequency. In addition, the minimum width of the beams was constrained by the width necessary to mount the strain gauges. A reasonable trade-off was arrived at by making use of the maximum sensitivity of the amplifier/recorder and determining a maximum allowable value for the cantilever beam length.

To prevent rotation of the cylinder, an offset arm was used to connect the horizontal force cantilever to the test module. A Farber Bearing, AMS 1K7, was used as a wheel at the test module end of the arm to reduce friction.

C. WAVE HEIGHT PROBE

A capacitance type probe was chosen to measure wave heights in this study. The probe essentially consists of a single insulated wire which acted as one plate of a capacitor, the water acting as the second plate. The capacitance varied as the water level rose and fell. A three-eighths inch diameter acrylic rod was attached to the bottom of the foil-like support member and the sensing wire was attached

at the bottom of the acrylic rod and insulated there. The tip of the acrylic rod was removable to provide easy changing of the sensing wire. The wire was connected at the top to a cable connector mounted on a nylon block. Number 30 A.W.G. wire with poly thermaleze insulation was used.

The schematic of the electronic circuit utilized with this probe is shown in Figure 9. In this circuit, the square wave output of a multi-vibrator is used to drive a capacitance bridge, one leg of which is varied by the wave height probe. The resulting AC error signal is converted to a D.C. output signal by the full wave rectifier. The output signal was found to be insensitive to small multivibrator frequency changes and linear through the range of the wave height probe. A model LD 5211-13 Brush D.C. amplifier was used with the probe circuit.

D. TEST PROCEDURE

Calibration of the probe was conducted with the tank filled to the desired level and the test module and wave height probe in position. With the probe immersed to a depth of nine inches, the amplifier gain was set at desired sensitivity and the recorder pen bias adjusted to position the recorder to mid-scale. The probe immersion was varied over a seven inch range in half inch increments, using a traverse mechanism. This initial calibration was checked at several points before and after each set of runs.

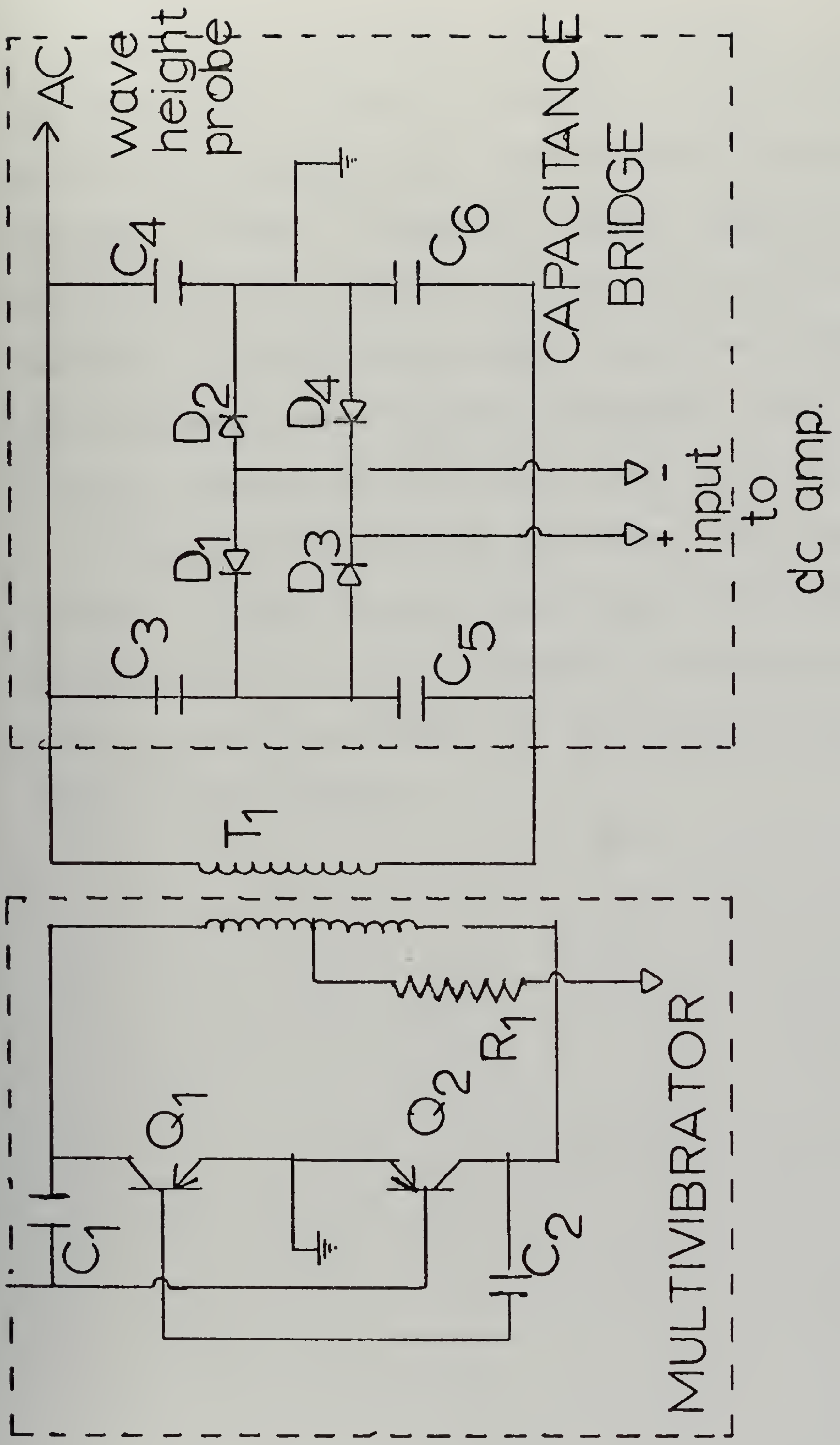


Fig. 9 WAVE HEIGHT PROBE CIRCUIT

The force cantilevers were calibrated by loading the cylinder with a series of weights, using a pulley arrangement shown in Figure 10. The load was transmitted to the cylinder through a series of tapped holes located around the circumference at positions horizontally fore and aft, and vertical. The Brush recorder output was calibrated by loading the cylinder in half-pound increments. This calibration was also checked prior to and after each series of runs.

After calibration was completed, the wave generator speed was set for the desired wave length and a data run commenced. During each run, wave generator eccentricity was varied from one inch to eight inches, or to a point where the waves broke in the channel.

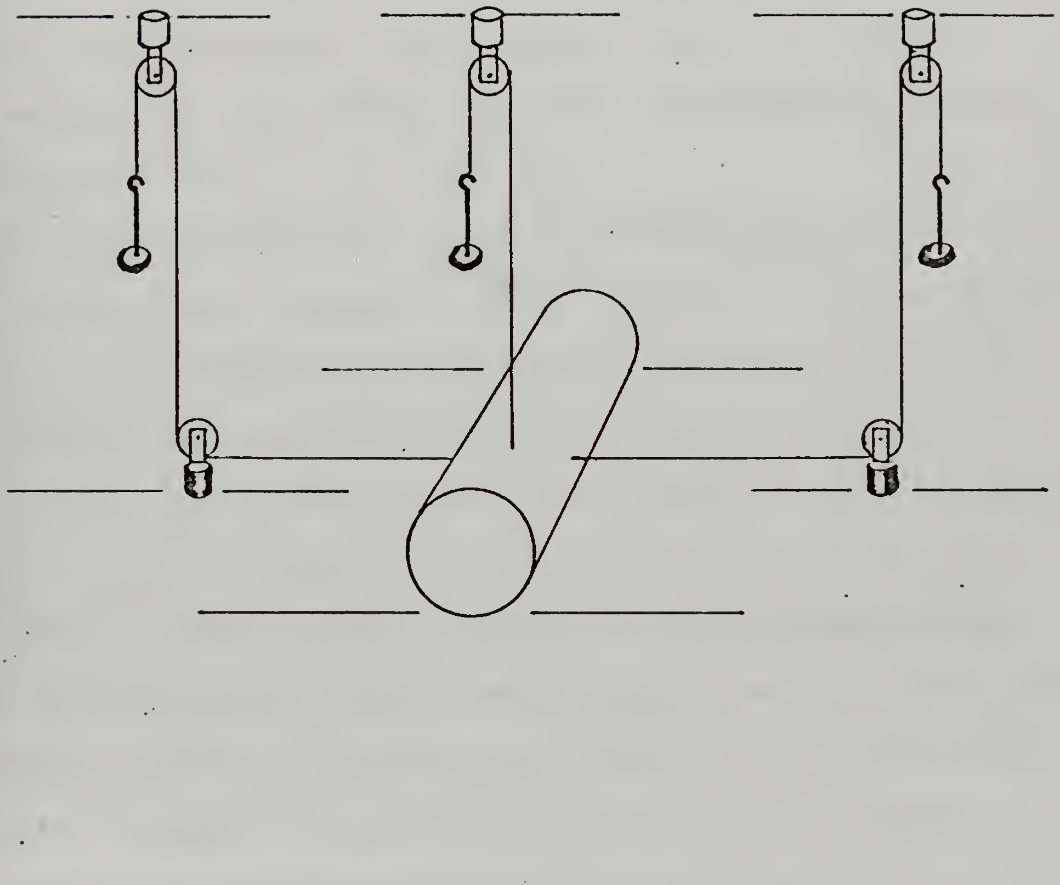


FIGURE 10.CANTILEVER CALIBRATION ARRANGEMENT

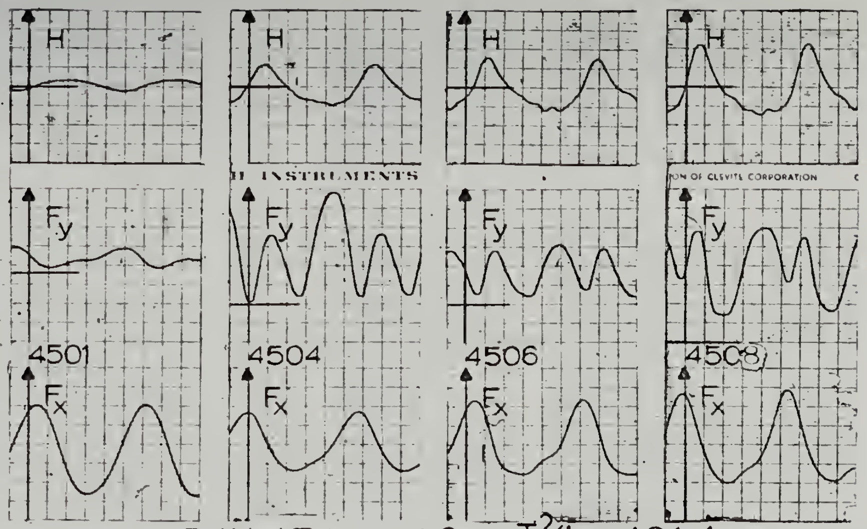
IV. PRESENTATION OF RESULTS AND CONCLUSIONS

As discussed in the theoretical analysis, dimensionless wave force coefficients can be represented as functions of the dimensionless water depth (h/a), the dimensionless wave height (H/2a), and dimensionless period (gT^2/h) parameters. A test program was developed which, for a given run, held water depth and period constant while varying wave height, in order to determine the effect of the dimensionless parameters on the horizontal and vertical force coefficients. Four series of experimental runs were conducted at relative water depths of 9.0, 7.0, 5.5, 4.0.

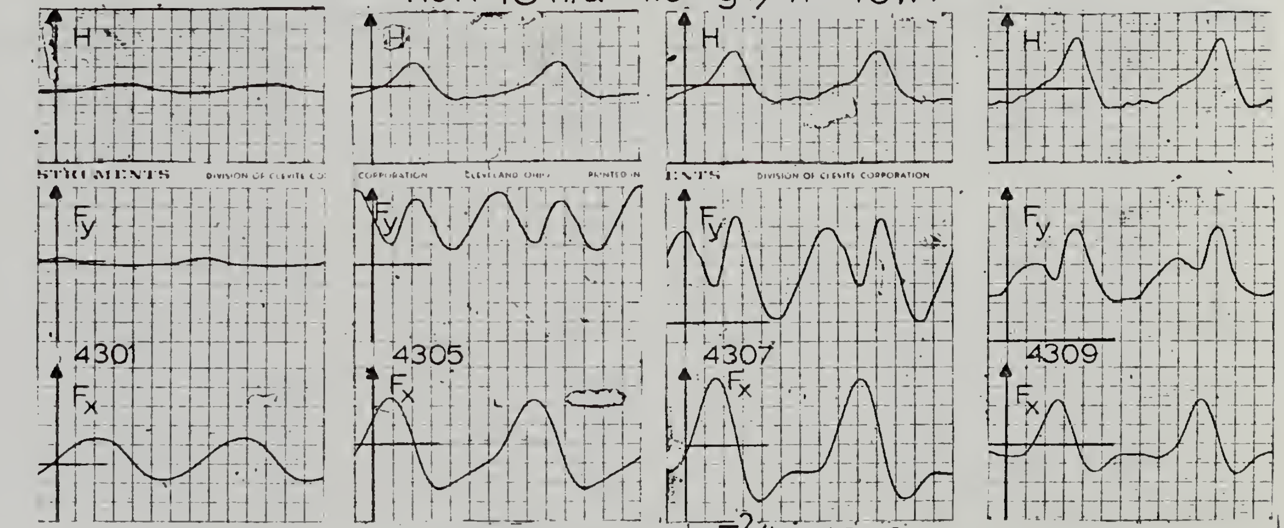
The dimensionless wave period parameter was varied for each series over a range of 100 to the maximum value attainable with the experimental apparatus. The upper limit in wave height was usually provided by breaking.

A series of representative traces are shown in Figures 11, 12, 13 and 14 due to the difficulty in presenting all the traces. These traces represent experimental data for three dimensionless periods from each of the four dimensionless water depths. The uppermost trace in each run represents wave height, the middle trace represents vertical force and the bottom trace represents horizontal force. The experimental values of the force coefficients for each experimental case are given in Appendix B.

Using the dimensionless period, water depth and waveheight as inputs, the computer program was used to generate



RUN 45 h/a = 4.0 $gT^2/h = 101.4$



RUN 43 h/a = 4.0 $gT^2/h = 171.9$

FIG. 11 Wave Height-Force Traces h/a = 4.0

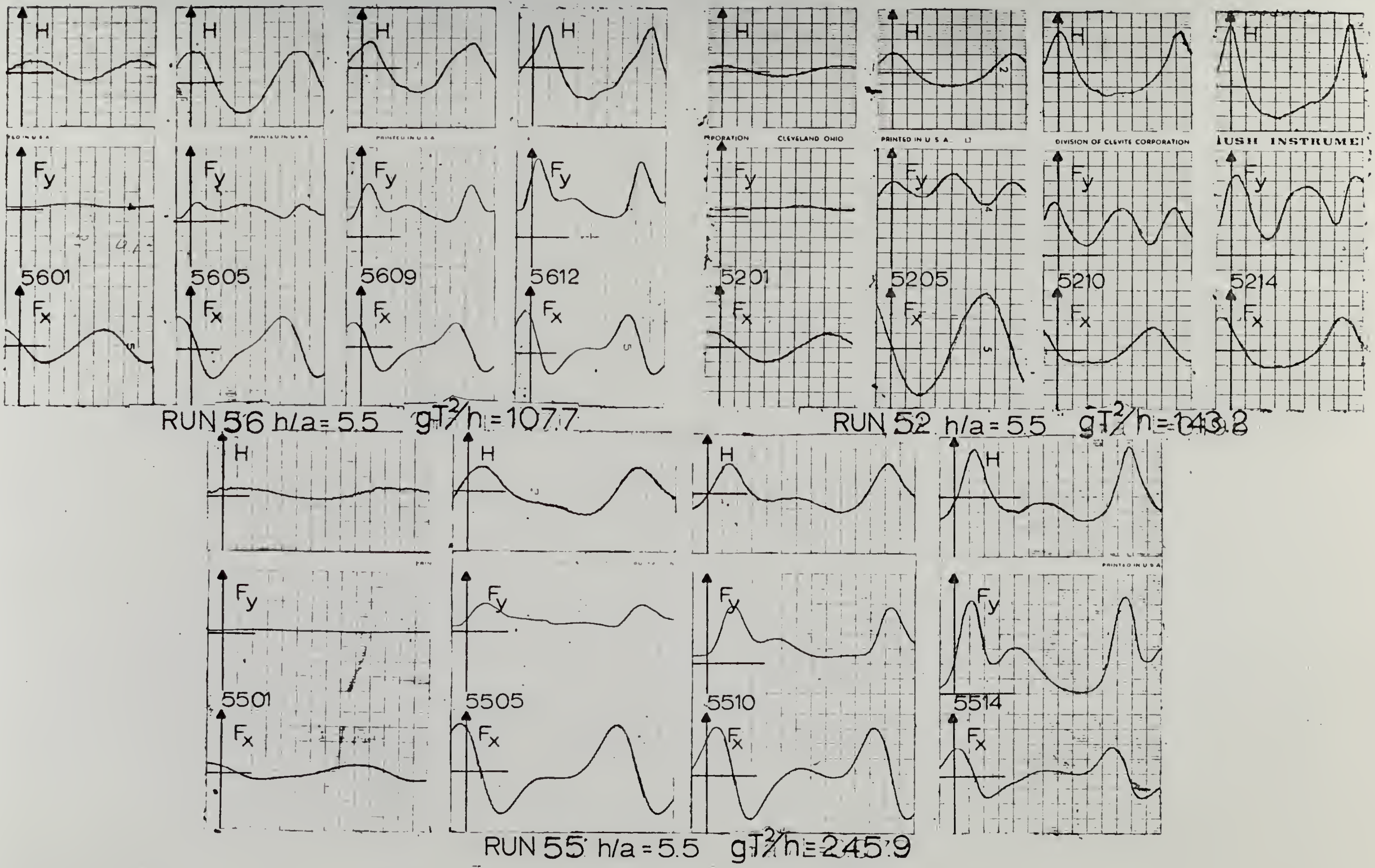


FIG. 12 Wave Height-Force Traces h/a = 5.5

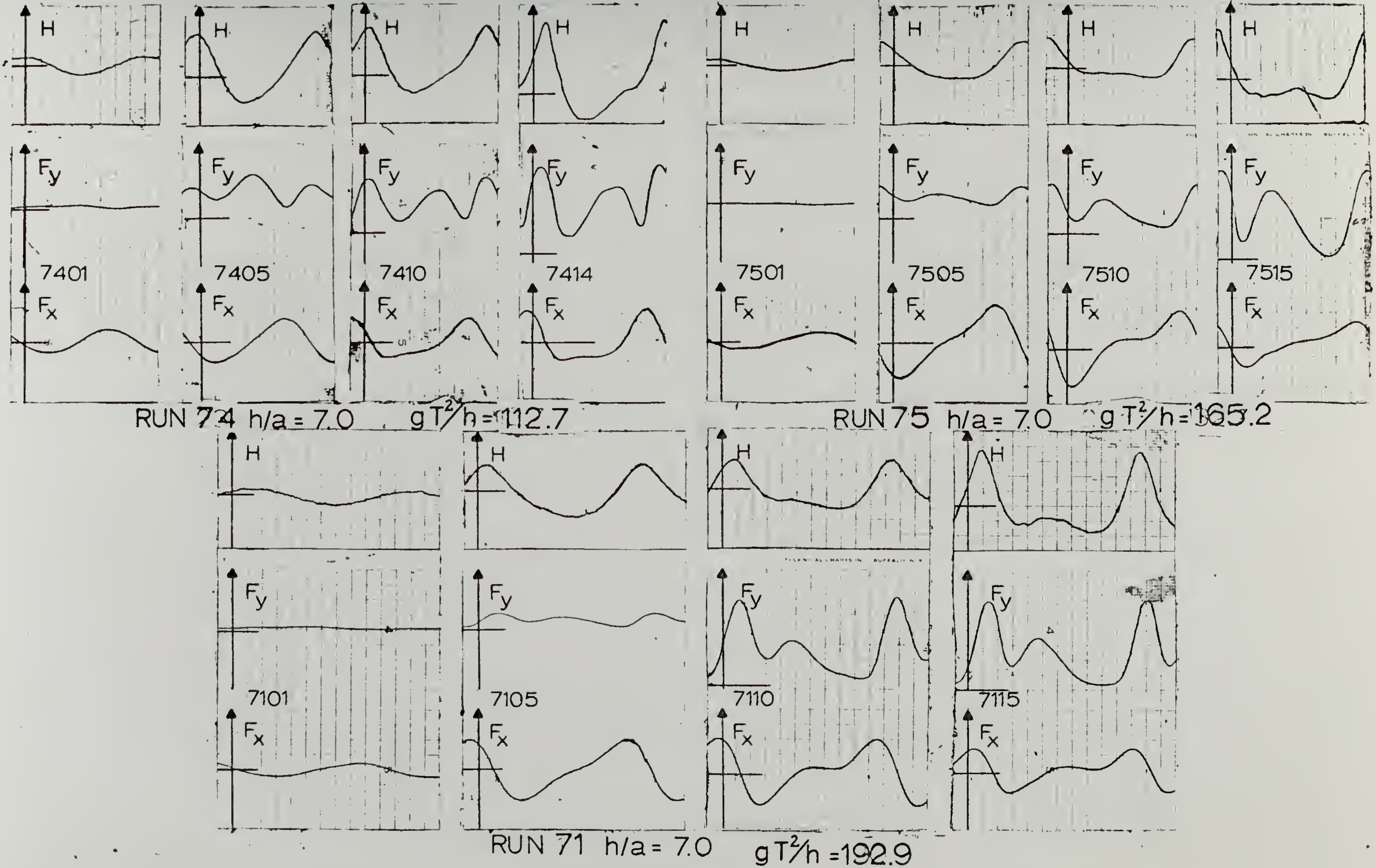


FIG. 13 Wave Height-Force Traces h/a = 7.0

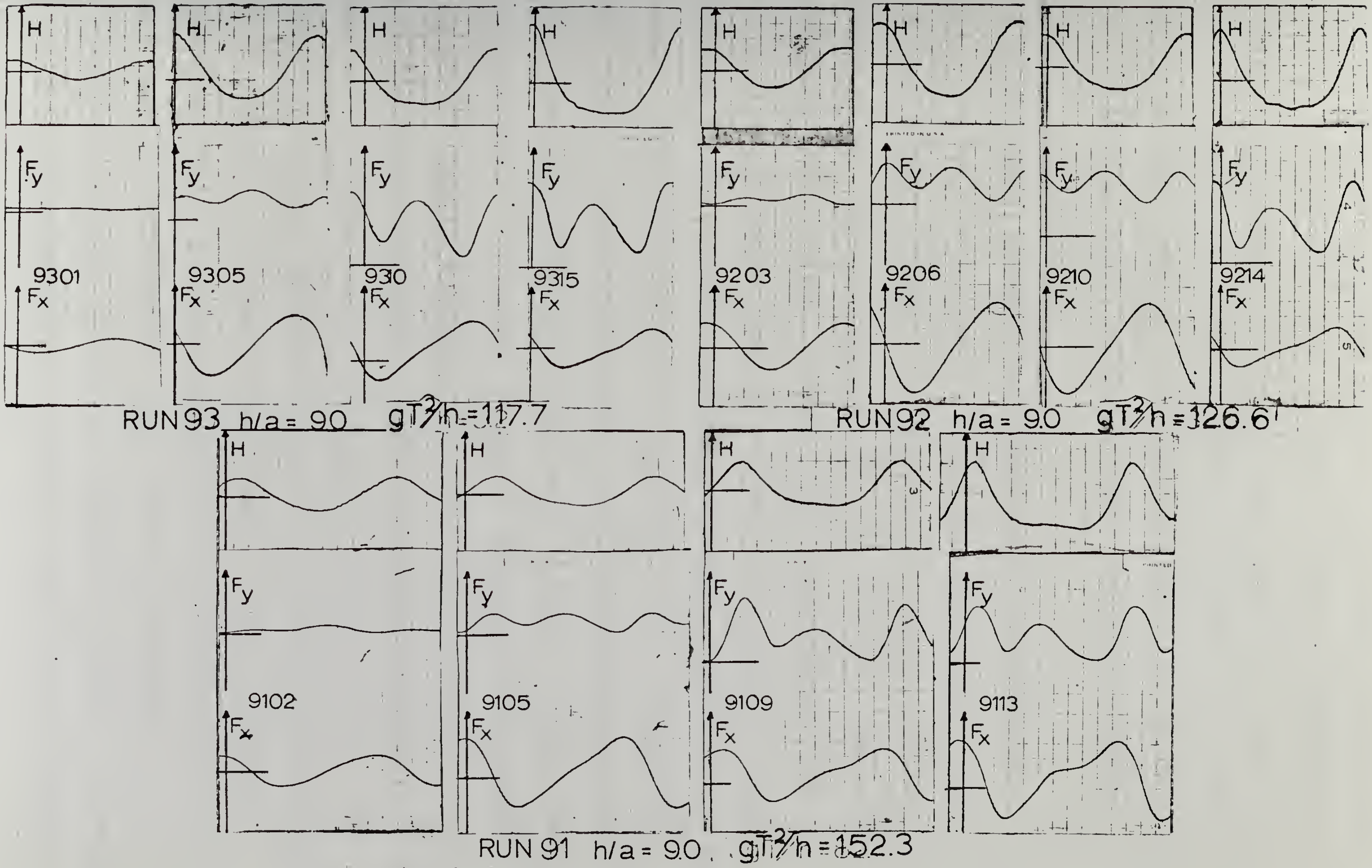


FIG. 14 Wave Height-Force Traces h/a = 9.0

corresponding theoretical values of the force coefficients for each experimental case. This provided a comparison between experimental and theoretical values for each run. Plots of experimental and theoretical values for maximum horizontal and vertical force coefficients as a function of wave height for each period and water depth studied are included in Appendix C.

Experimental results and theoretical computations indicate that, at the lower values of dimensionless period and wave height studied, the wave may be characterized as a linear wave. The horizontal force was found to vary sinusoidally, and increased linearly with waveheight. The vertical force varied approximately as a negative cosine function in this region, having its maximum value in the wave trough. The magnitude of the vertical force coefficient was found to vary approximately as the waveheight squared. These characteristics are shown most clearly in runs Number 44 through 47, 71 through 74 and 93 through 95 in Appendix C.

As the relative period and wave height increased, the wave displayed the sharper peaks and longer, shallower troughs that are characteristic of non-linear gravity waves. The horizontal force, proportional to the fluid particle acceleration, tended to reach maximum values closer to the wave peaks, but still remained symmetrical, with respect to the wave crest, as indicated by the theoretical traces. The theoretical analysis predicted a phase lag between the wave crest and the maximum force as small as 24° at the higher wave heights.

The vertical force component began to display positive values occurring at twice the frequency of the wave peaks, as shown in the experimental traces, Figures 11, 12, 13, 14. This is explained by the increasing importance of the lift component of vertical force, which, theoretically, is similar to a cosine squared function. At extremely long periods, the lift force is clearly dominant, and the maximum force no longer occurs in the middle of the wave trough. The maximum vertical force occurs when the cylinder is under the wave crests where the velocity is greatest.

At large periods and wave heights, a point is reached where flow separation effects are no longer negligible. This is seen most clearly in the vertical force component, whose value becomes suddenly lower than the theory predicts, though still varying as the waveheight squared. This sudden drop in lift is apparently caused by a corresponding decrease in the lift coefficient resulting from flow separation. This phenomenon is most clearly evident in the graphical presentation of runs 51 through 56, and 71 through 73 in Appendix C.

1). Horizontal Forces

As previously mentioned, the horizontal forces tended to be nearly sinusoidal in nature with a maximum value which is quite linear with wave height. It is therefore possible to characterize the horizontal force coefficient by it's maximum value ($F_{x_{max}}/\rho g a^2 \ell$). The variation in the horizontal force coefficient with dimensionless wave height is shown graphically in Appendix C.

The experimental values of horizontal force are seen to vary linearly with wave height for relative period parameter values from 100 to 120. For relative period values greater than 120, the force coefficient varies in an increasingly non-linear fashion. This is consistent with the theory, as shown in Figure 15. However, for the range of period parameters studied, the deviation from linearity is relatively small. It is therefore possible to display the experimental data in terms of the slope of the force coefficient versus wave height plots, as shown in Figures 16 and 17.

The theory predicts the maximum values of horizontal force coefficients prior to the wave heights where separation effects become apparent. The discrepancy between theoretical and experimental values is due to variation of the added mass coefficient in a separated flow as well as a net drag force which is no longer zero.

2). Vertical Forces

As indicated previously, the vertical force displays two regimes; one inertia dominated and one lift dominated. The maximum value of the vertical force coefficient $(F_{y_{max}}/\rho g a^2 \ell)$ is seen to vary as the wave height squared throughout the range of data taken. This is shown graphically in Appendix C. For higher values of relative wave height, the magnitude of the vertical force coefficients is less than predicted by the theory, due to separation effects, as explained above.

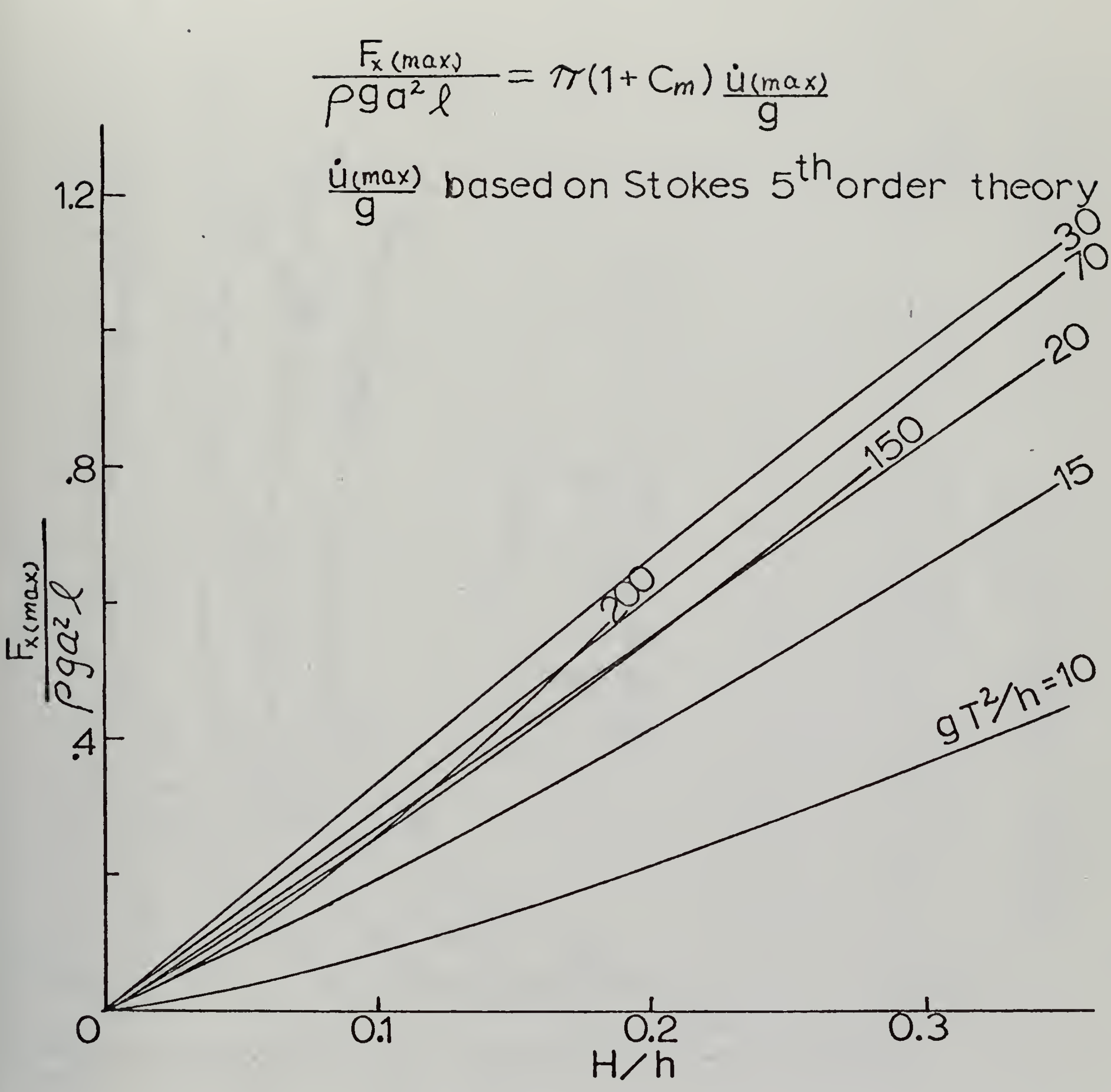


Fig. 15 HORIZ. WAVE FORCE

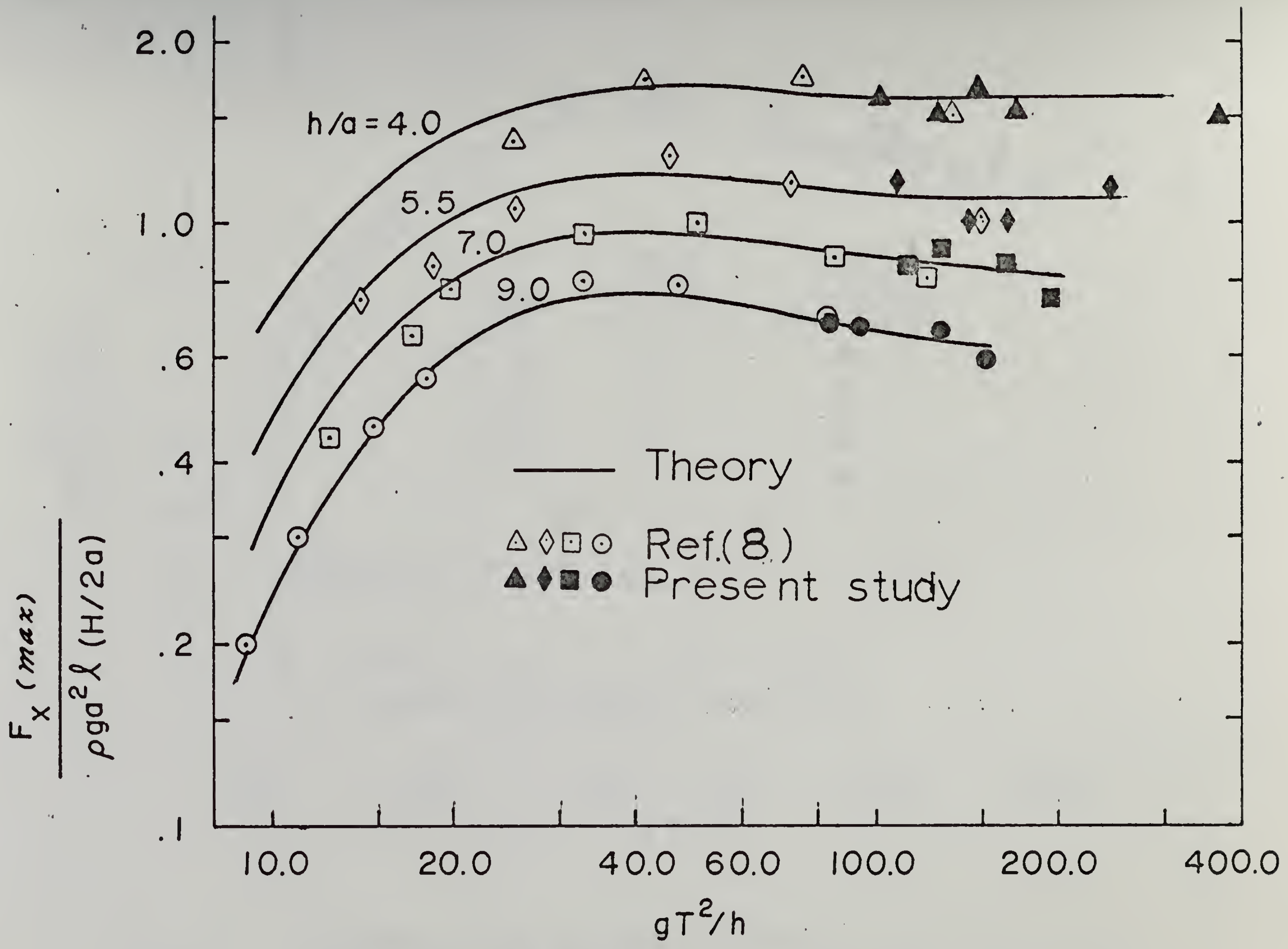


Fig.16 HORIZONTAL WAVE FORCE

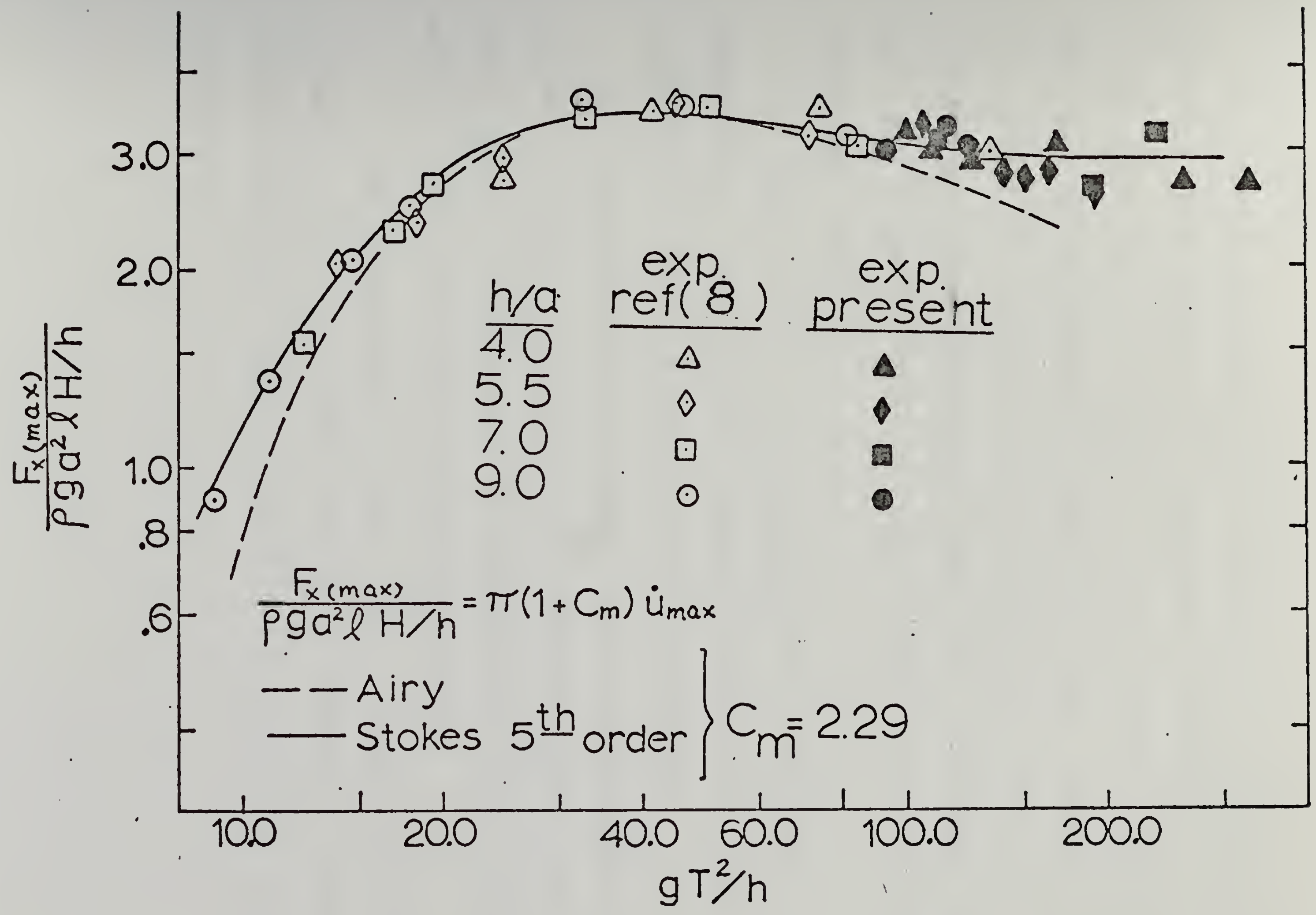


Fig. 17 HORIZONTAL WAVE FORCE

B. CONCLUSIONS

From the experimental and theoretical results, the following conclusions are considered warranted:

1. The horizontal forces on the cylinder, resulting from the incident gravity waves, increased linearly with wave height for relative period values less than 120. For relative period values greater than 120, the force coefficient was found to vary with wave height in an increasingly non-linear fashion.

2. The vertical forces displayed two regimes; one inertia dominated at lower periods and wave heights, and one lift dominated at higher values of these parameters.

3. For high period parameter values, separation effects can be expected for relative wave height values above 0.6, resulting in a sudden drop in experimental force coefficient values from those predicted by the theory.

4. The theory used in this study provides excellent agreement with experimental values for relative period (gT^2/h) values from 100 to 200, and prior to the onset of flow separation.

APPENDIX A

COEFFICIENTS FOR STOKES FIFTH ORDER WAVE THEORY

$$S = \sinh(\tilde{d})$$

$$C = \cosh(\tilde{d})$$

$$C_0 = \tanh(\tilde{d})$$

$$C_1 = \frac{(8C^4-8C^2+9)}{8S^4}$$

$$C_2 = \frac{(3840C^{12}-4096C^{10}+2592C^8-1008C^6+5944C^4-1830C^2+147)}{512S^{10}(6C^2-1)}$$

$$C_3 = -\frac{1}{4SC}$$

$$C_4 = \frac{(12C^8+36C^6-162C^4+141C^2-27)}{192CS^9}$$

$$A_{11} = \frac{1}{S}$$

$$A_{13} = \frac{-C^2(5C^2+1)}{8S^5}$$

$$A_{15} = \frac{-(1184C^{10}-1440C^8-1992C^6+2641C^4-249C^2+18)}{1536S^{11}}$$

$$A_{22} = \frac{3}{8S^4}$$

$$A_{24} = \frac{(192C^8-424C^6-312C^4+480C^2-17)}{768S^{10}}$$

$$A_{33} = \frac{(13-4C^2)}{64S^7}$$

$$A_{35} = \frac{(512C^{12}+4224C^{10}-6800C^8-12,808C^6+16,704C^4-3154C^2+107)}{4096S^{13}(6C^2-1)}$$

$$A_{44} = \frac{(80C^6-816C^4+1338C^2-197)}{1536S^{10}(6C^2-1)}$$

$$A_{55} = \frac{-(2880C^{10}-72,480C^8+324,000C^6-432,000C^4+163,470C^2-16,245)}{61,440S^{11}(6C^2-1)(8C^4-11C^2+3)}$$

$$B_{22} = \frac{(2C^2+1)}{4S^3}$$

$$B_{24} = \frac{C(272C^8-504C^6-192C^4+322C^2+21)}{384S^9}$$

$$B_{33} = \frac{3(8C^6+1)}{63S^6}$$

$$B_{35} = \frac{(88,128C^{14}-208,224C^{12}+70,818C^{10}+54,000C^8-21,816C^6+6264C^4-54C^2-81)}{12,228S^{12}(6C^2-1)}$$

$$B_{44} = \frac{C(768C^{10}-448C^8-48C^6+48C^4+106C^2-21)}{384S^9(6C^2-1)}$$

$$B_{55} = \frac{(192,000C^{16}-262,720C^{14}+83,680C^{12}+20,160C^{10}-7280C^8)}{12,288S^{10}(6C^2-1)(8C^4-11C^2+3)} + \\ + \frac{(7160C^6-1800C^4-1050C^2+225)}{12,288S^{10}(6C^2-1)(8C^4-11C^2+3)}$$

$$A_1 = \lambda A_{11} + \lambda^3 A_{13} + \lambda^5 A_{15}$$

$$A_2 = \lambda^2 A_{22} + \lambda^4 A_{24}$$

$$A_3 = \lambda^3 A_{33} + \lambda^5 A_{35}$$

$A_4 = \lambda^4 A_{44}$

$A_5 = \lambda^5 A_{55}$

$B_2 = \lambda^2 B_{22} + \lambda^4 B_{24}$

$B_3 = \lambda^3 B_{33} + \lambda^5 B_{35}$

$B_4 = \lambda^4 B_{44}$

$B_5 = \lambda^5 B_{55}$

APPENDIX B

EXPERIMENTAL DATA

```
DATA FOR RUN NUMBER 41.  TOTAL NUMBER OF RUNS IS 23.

WAVE PERIOD IS  2.637SECONDS.    WATER DEPTH IS  8.0INCHES.

DIMENSIONLESS PARAMETERS ARE   NON-DIMENSIONAL PERIOD(=GT**2/H) IS335.681
                                     DEPTH TO RADIUS RATIO(=H/A) IS 4.0000



SEGMENT   WAVE     FORCE    FORCE    H/2A    F. COEF    F. COEF
NUMBER    HEIGHT    (X)       (Y)            (X)          (Y)


  4101    0.30     0.23     0.03    0.08     0.11       0.01
  4102    0.51     0.56     0.05    0.13     0.26       0.03
  4103    0.63     0.62     0.12    0.16     0.29       0.05
  4104    1.01     0.96     0.23    0.25     0.44       0.11
  4105    1.21     1.33     0.34    0.30     0.62       0.16
  4106    1.34     1.71     0.48    0.33     0.79       0.22
  4107    1.62     1.71     0.59    0.40     0.79       0.27
  4108    1.86     1.81     0.80    0.46     0.84       0.37
  4109    2.46     2.20     0.94    0.62     1.02       0.44
  4110    2.60     2.37     1.05    0.65     1.10       0.49
  4111    2.87     2.56     1.27    0.72     1.18       0.59
```

```
DATA FOR RUN NUMBER 42.  TOTAL NUMBER OF RUNS IS 23.

WAVE PERIOD IS  2.160SECONDS.    WATER DEPTH IS  8.0INCHES.

DIMENSIONLESS PARAMETERS ARE   NON-DIMENSIONAL PERIOD(=GT**2/H) IS225.138
                                        DEPTH TO RADIUS RATIO(=H/A) IS 4.0000



SEGMENT    WAVE      FORCE     FORCE      H/2A     F. COEF    F. COEF
NUMBER     HEIGHT     (X)        (Y)               (X)          (Y)


  4201      0.31      0.25      0.04      0.08       0.12      0.02
  4202      0.50      0.50      0.06      0.12       0.23      0.03
  4203      0.83      0.63      0.11      0.21       0.29      0.05
  4204      1.13      0.83      0.19      0.28       0.39      0.09
  4205      1.47      1.05      0.30      0.37       0.49      0.14
  4206      1.91      1.34      0.37      0.48       0.62      0.17
  4207      2.13      1.49      0.57      0.53       0.69      0.26
  4208      2.47      1.76      0.76      0.62       0.82      0.35
  4209      2.83      1.77      0.94      0.71       0.82      0.44
  4210      3.13      1.92      1.03      0.78       0.89      0.48
  4211      3.39      2.12      1.18      0.85       0.98      0.55
```

```
DATA FOR RUN NUMBER 43.  TOTAL NUMBER OF RUNS IS 23.

WAVE PERIOD IS  1.887SECONDS.     WATER DEPTH IS  8.0INCHES.

DIMENSIONLESS PARAMETERS ARE   NON-DIMENSIONAL PERIOD(=GT**2/H) IS171.916
                                   DEPTH TO RADIUS RATIO(=H/A) IS 4.0000



SEGMENT    WAVE      FORCE     FORCE      H/2A     F. COEF    F. COEF
NUMBER     HEIGHT     (X)        (Y)               (X)         (Y)

  4301      0.36      0.28      0.04      0.09      0.13       0.02
  4302      0.60      0.53      0.12      0.15      0.25       0.06
  4303      0.89      0.74      0.19      0.22      0.34       0.09
  4304      1.07      0.82      0.26      0.27      0.38       0.12
  4305      1.40      1.23      0.36      0.35      0.57       0.17
  4306      1.71      1.52      0.52      0.43      0.70       0.24
  4307      2.06      1.81      0.68      0.52      0.84       0.32
  4308      2.38      2.26      0.89      0.60      1.05       0.41
  4309      2.57      2.36      0.97      0.64      1.09       0.45
  4310      2.82      2.79      1.13      0.70      1.29       0.52
```

```
DATA FOR RUN NUMBER 44.  TOTAL NUMBER OF RUNS IS 23.

WAVE PERIOD IS  1.637SECONDS.    WATER DEPTH IS  8.0INCHES.

DIMENSIONLESS PARAMETERS ARE   NON-DIMENSIONAL PERIOD(=GT**2/H) IS129.391
                                      DEPTH TO RADIUS RATIO(=H/A) IS 4.0000



SEGMENT    WAVE      FORCE    FORCE     H/2A    F. COEF   F. COEF
NUMBER     HEIGHT     (X)      (Y)              (X)        (Y)

  4401      0.46      0.25     0.05     0.12     0.12      0.02
  4402      0.78      0.53     0.12     0.20     0.25      0.06
  4403      1.08      0.88     0.20     0.27     0.41      0.09
  4404      1.35      1.17     0.25     0.34     0.54      0.12
  4405      1.78      1.29     0.37     0.44     0.60      0.17
  4406      2.23      1.75     0.57     0.56     0.81      0.27
  4407      2.36      2.11     0.70     0.59     0.98      0.32
  4408      2.74      2.26     0.86     0.69     1.05      0.40
  4409      3.04      2.58     1.03     0.76     1.19      0.47
```

```
DATA FOR RUN NUMBER 45.  TOTAL NUMBER OF RUNS IS 23.

WAVE PERIOD IS  1.450SECONDS.     WATER DEPTH IS  8.0INCHES.

DIMENSIONLESS PARAMETERS ARE    NON-DIMENSIONAL PERIOD(=GT**2/H) IS101.456
                                        DEPTH TO RADIUS RATIO(=H/A) IS 4.0000



SEGMENT    WAVE     FORCE    FORCE    H/2A     F. COEF   F. COEF
NUMBER     HEIGHT    (X)      (Y)              (X)        (Y)

 4501      0.44     0.58     0.10     0.11      0.27     0.05
 4502      0.75     0.59     0.23     0.19      0.27     0.10
 4503      1.06     1.49     0.20     0.27      0.69     0.09
 4504      1.39     1.73     0.37     0.35      0.80     0.17
 4505      1.73     1.86     0.43     0.43      0.86     0.20
 4506      2.09     1.93     0.49     0.52      0.89     0.22
 4507      2.64     2.26     0.62     0.66      1.05     0.29
 4508      3.07     2.66     0.78     0.77      1.23     0.36
```

```
DATA FOR RUN NUMBER 46.  TOTAL NUMBER OF RUNS IS 23.

WAVE PERIOD IS  1.755SECONDS.    WATER DEPTH IS  8.0INCHES.

DIMENSIONLESS PARAMETERS ARE   NON-DIMENSIONAL PERIOD(=GT**2/H) IS148.626
                                    DEPTH TO RADIUS RATIO(=H/A) IS 4.0000



SEGMENT    WAVE      FORCE     FORCE     H/2A     F. COEF    F. COEF
NUMBER     HEIGHT     (X)       (Y)               (X)         (Y)

  4601      0.47      0.34      0.06      0.12      0.16      0.03
  4602      0.79      0.56      0.11      0.20      0.26      0.05
  4603      1.12      0.92      0.19      0.28      0.43      0.09
  4604      1.36      1.04      0.17      0.34      0.48      0.08
  4605      1.97      1.67      0.39      0.49      0.77      0.18
  4606      2.22      2.05      0.54      0.55      0.95      0.25
  4607      2.64      2.47      0.72      0.66      1.14      0.33
  4608      3.04      2.71      0.91      0.76      1.25      0.42
  4609      3.58      2.98      0.95      0.89      1.38      0.44
  4610      3.68      2.83      1.05      0.92      1.31      0.49
```

```
DATA FOR RUN NUMBER 47.  TOTAL NUMBER OF RUNS IS 23.

WAVE PERIOD IS  1.535SECONDS.    WATER DEPTH IS  8.0INCHES.

DIMENSIONLESS PARAMETERS ARE    NON-DIMENSIONAL PERIOD(=GT**2/H) IS113.700
                                        DEPTH TO RADIUS RATIO(=H/A) IS 4.0000



SEGMENT    WAVE      FORCE    FORCE     H/2A    F. COEF   F. COEF
NUMBER     HEIGHT     (X)       (Y)              (X)        (Y)

  4701      0.50      0.54     0.08     0.12      0.25      0.04
  4702      0.92      0.83     0.10     0.23      0.39      0.05
  4703      1.23      1.15     0.22     0.31      0.53      0.10
  4704      1.71      1.51     0.30     0.43      0.70      0.14
  4705      1.96      1.72     0.32     0.49      0.80      0.15
  4706      2.35      1.99     0.69     0.59      0.92      0.32
  4707      2.73      2.26     0.69     0.68      1.05      0.32
  4708      3.20      2.83     0.74     0.80      1.31      0.34
  4709      3.36      2.93     1.01     0.84      1.35      0.47
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```
DATA FOR RUN NUMBER 51.  TOTAL NUMBER OF RUNS IS 23.

WAVE PERIOD IS  2.175SECONDS.    WATER DEPTH IS 11.0INCHES.

DIMENSIONLESS PARAMETERS ARE   NON-DIMENSIONAL PERIOD(=GT**2/H) IS166.019
                                          DEPTH TO RADIUS RATIO(=H/A) IS 5.5000



SEGMENT   WAVE     FORCE    FORCE     H/2A     F. COEF    F. COEF
NUMBER    HEIGHT    (X)      (Y)               (X)         (Y)

  5101    0.49     0.20     0.01     0.12      0.09       0.01
  5102    0.67     0.32     0.05     0.17      0.15       0.02
  5103    0.91     0.47     0.09     0.23      0.22       0.04
  5104    1.27     0.62     0.15     0.32      0.29       0.07
  5105    1.56     0.77     0.21     0.39      0.36       0.10
  5106    1.85     0.96     0.34     0.46      0.44       0.16
  5107    2.05     1.14     0.51     0.51      0.53       0.23
  5108    2.44     1.29     0.66     0.61      0.60       0.30
  5109    2.85     1.45     0.85     0.71      0.67       0.40
  5110    3.23     1.51     0.89     0.81      0.70       0.41
  5111    3.62     1.75     0.92     0.91      0.81       0.43
  5112    3.96     1.99     1.12     0.99      0.92       0.52
  5113    4.41     2.11     1.28     1.10      0.97       0.59
  5114    4.73     2.33     1.61     1.18      1.08       0.74
  5115    4.99     2.51     1.88     1.25      1.16       0.87
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```
DATA FOR RUN NUMBER 52.  TOTAL NUMBER OF RUNS IS 23.

WAVE PERIOD IS  2.020SECONDS.    WATER DEPTH IS 11.0INCHES.

DIMENSIONLESS PARAMETERS ARE   NON-DIMENSIONAL PERIOD(=GT**2/H) IS143.200
                                       DEPTH TO RADIUS RATIO(=H/A) IS 5.5000



SEGMENT    WAVE      FORCE    FORCE     H/2A    F. COEF   F. COEF
NUMBER     HEIGHT     (X)       (Y)             (X)         (Y)

  5201      0.47      0.20     0.03     0.12     0.09      0.02
  5202      0.73      0.34     0.07     0.18     0.16      0.03
  5203      0.99      0.53     0.13     0.25     0.25      0.06
  5204      1.55      0.66     0.23     0.39     0.30      0.11
  5205      1.65      0.82     0.32     0.41     0.38      0.15
  5206      2.02      1.04     0.46     0.50     0.48      0.21
  5207      2.41      1.25     0.59     0.60     0.58      0.27
  5208      2.61      1.53     0.74     0.65     0.71      0.34
  5209      3.04      1.78     0.87     0.76     0.82      0.40
  5210      3.46      2.04     0.95     0.87     0.94      0.44
  5211      3.75      2.31     1.14     0.94     1.07      0.53
  5212      4.15      2.64     1.54     1.04     1.22      0.71
  5213      4.62      2.86     1.47     1.16     1.32      0.68
  5214      5.14      3.05     1.65     1.28     1.41      0.76
```

```
DATA FOR RUN NUMBER 53.  TOTAL NUMBER OF RUNS IS 23.

WAVE PERIOD IS  1.880SECONDS.    WATER DEPTH IS 11.0INCHES.

DIMENSIONLESS PARAMETERS ARE   NON-DIMENSIONAL PERIOD(=GT**2/H) IS124.038
                                  DEPTH TO RADIUS RATIO(=H/A)  IS 5.5000



SEGMENT   WAVE     FORCE   FORCE    H/2A   F. COEF  F. COEF
NUMBER    HEIGHT    (X)     (Y)             (X)       (Y)

  5301     0.40     0.27    0.04    0.10    0.12     0.02
  5302     0.63     0.44    0.09    0.16    0.21     0.04
  5303     0.92     0.62    0.14    0.23    0.28     0.06
  5304     1.25     0.81    0.24    0.31    0.37     0.11
  5305     1.46     1.09    0.30    0.36    0.51     0.14
  5306     1.80     1.35    0.39    0.45    0.62     0.18
  5307     2.06     1.57    0.55    0.52    0.72     0.25
  5308     2.36     1.80    0.75    0.59    0.83     0.35
  5309     2.72     1.86    0.98    0.68    0.86     0.45
  5310     3.10     2.26    1.10    0.77    1.05     0.51
  5311     3.51     2.56    1.21    0.88    1.18     0.56
  5312     3.72     2.86    1.36    0.93    1.32     0.63
  5313     4.25     3.12    1.54    1.06    1.44     0.71
  5314     4.50     3.26    1.67    1.13    1.51     0.77
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DATA FOR RUN NUMBER 54.  TOTAL NUMBER OF RUNS IS 23.

WAVE PERIOD IS  2.322SECONDS.    WATER DEPTH IS 11.0INCHES.

DIMENSIONLESS PARAMETERS ARE   NON-DIMENSIONAL PERIOD(=GT**2/H) IS189.300
                                         DEPTH TO RADIUS RATIO(=H/A) IS 5.5000



SEGMENT   WAVE     FORCE    FORCE     H/2A     F. COEF   F. COEF
NUMBER    HEIGHT    (X)      (Y)               (X)        (Y)

  5401    0.31     0.11     0.01      0.08      0.05      0.00
  5402    0.50     0.31     0.04      0.13      0.14      0.02
  5403    0.71     0.45     0.08      0.18      0.21      0.04
  5404    0.93     0.59     0.14      0.23      0.27      0.07
  5405    1.14     0.73     0.21      0.29      0.34      0.10
  5406    1.43     0.97     0.23      0.36      0.45      0.11
  5407    1.72     1.08     0.34      0.43      0.50      0.16
  5408    2.01     1.30     0.45      0.50      0.60      0.21
  5409    2.30     1.51     0.47      0.57      0.70      0.22
  5410    2.54     1.57     0.47      0.63      0.72      0.22
  5411    3.26     1.50     0.65      0.81      0.69      0.30
  5412    3.52     1.55     0.92      0.88      0.72      0.43
  5413    3.88     1.75     1.10      0.97      0.81      0.51
  5414    4.34     1.88     1.17      1.09      0.87      0.54
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```
DATA FOR RUN NUMBER 55.  TOTAL NUMBER OF RUNS IS 23.

WAVE PERIOD IS  2.647SECONDS.    WATER DEPTH IS 11.0INCHES.

DIMENSIONLESS PARAMETERS ARE   NON-DIMENSIONAL PERIOD(=GT**2/H) IS245.986
                                    DEPTH TO RADIUS RATIO(=H/A) IS 5.5000



SEGMENT    WAVE      FORCE     FORCE     H/2A     F. COEF    F. COEF
NUMBER     HEIGHT     (X)       (Y)               (X)        (Y)


  5501     0.30      0.10      0.01     0.07      0.05       0.00
  5502     0.41      0.26      0.03     0.10      0.12       0.01
  5503     0.67      0.43      0.03     0.17      0.20       0.02
  5504     0.75      0.60      0.10     0.19      0.28       0.05
  5505     1.05      0.75      0.19     0.26      0.35       0.09
  5506     1.37      0.84      0.29     0.34      0.39       0.13
  5507     1.76      1.09      0.29     0.44      0.50       0.13
  5508     2.06      1.20      0.55     0.51      0.55       0.26
  5509     2.32      1.36      0.75     0.58      0.63       0.35
  5510     2.69      1.58      0.78     0.67      0.73       0.36
  5511     3.17      1.71      0.92     0.79      0.79       0.43
  5512     3.34      1.77      0.98     0.84      0.82       0.45
  5513     3.70      1.94      1.21     0.92      0.90       0.56
  5514     4.17      2.17      1.40     1.04      1.01       0.65
```

```
DATA FOR RUN NUMBER 56.  TOTAL NUMBER OF RUNS IS 23.

WAVE PERIOD IS  1.752SECONDS.    WATER DEPTH IS 11.0INCHES.

DIMENSIONLESS PARAMETERS ARE   NON-DIMENSIONAL PERIOD(=GT**2/H) IS107.784
                                      DEPTH TO RADIUS RATIO(=H/A) IS 5.5000



SEGMENT   WAVE     FORCE    FORCE     H/2A     F. COEF   F. COEF
NUMBER    HEIGHT    (X)       (Y)               (X)        (Y)


  5601     0.56     0.28     0.02     0.14      0.13      0.01
  5602     0.94     0.51     0.07     0.23      0.24      0.03
  5603     1.05     0.74     0.13     0.26      0.34      0.06
  5604     1.20     0.93     0.23     0.30      0.43      0.11
  5605     1.66     1.11     0.36     0.42      0.51      0.17
  5606     2.11     1.31     0.40     0.53      0.61      0.18
  5507     2.46     1.55     0.56     0.62      0.72      0.26
  5608     2.82     1.63     0.76     0.70      0.76      0.35
  5609     3.17     1.88     0.77     0.79      0.87      0.36
  5610     3.43     2.06     0.85     0.86      0.95      0.39
  5611     3.78     2.45     0.92     0.95      1.13      0.43
  5612     4.66     2.55     1.03     1.17      1.18      0.48
```

```
DATA FOR RUN NUMBER 71.  TOTAL NUMBER OF RUNS IS 23.

WAVE PERIOD IS  2.645SECONDS.    WATER DEPTH IS 14.0INCHES.

DIMENSIONLESS PARAMETERS ARE   NON-DIMENSIONAL PERIOD(=GT**2/H) IS192.910
                                   DEPTH TO RADIUS RATIO(=H/A) IS 7.0000




SEGMENT   WAVE     FORCE   FORCE     H/2A    F. COEF   F. COEF
NUMBER    HEIGHT    (X)     (Y)               (X)       (Y)


  7101     0.30     0.10    0.02     0.07     0.05      0.01
  7102     0.60     0.21    0.03     0.15     0.10      0.02
  7103     0.82     0.27    0.05     0.21     0.13      0.02
  7104     1.09     0.40    0.08     0.27     0.18      0.04
  7105     1.27     0.56    0.16     0.32     0.26      0.07
  7106     1.57     0.68    0.22     0.39     0.31      0.10
  7107     1.94     0.78    0.36     0.48     0.36      0.16
  7108     2.24     0.93    0.45     0.56     0.43      0.21
  7109     2.64     1.06    0.62     0.66     0.49      0.29
  7110     2.83     1.20    0.77     0.71     0.55      0.36
  7111     3.26     1.33    0.95     0.81     0.62      0.44
  7112     3.87     1.42    0.93     0.97     0.66      0.43
  7113     4.05     1.50    0.97     1.01     0.69      0.45
  7114     4.40     1.72    1.24     1.10     0.80      0.57
  7115     4.75     1.88    1.42     1.19     0.87      0.66
```

```
DATA FOR RUN NUMBER720.  TOTAL NUMBER OF RUNS IS 23.

WAVE PERIOD IS  2.365SECONDS.    WATER DEPTH IS 14.0INCHES.

DIMENSIONLESS PARAMETERS ARE   NON-DIMENSIONAL PERIOD(=GT**2/H) IS154.229
                                       DEPTH TO RADIUS RATIO(=H/A) IS 7.0000



SEGMENT   WAVE     FORCE    FORCE    H/2A    F. COEF    F. COEF
NUMBER    HEIGHT    (X)      (Y)             (X)         (Y)

  7201    0.34     0.16     0.01     0.08     0.07      0.01
  7202    0.60     0.16     0.03     0.15     0.07      0.01
  7203    0.79     0.29     0.06     0.20     0.13      0.03
  7204    0.99     0.34     0.08     0.25     0.16      0.04
  7205    1.27     0.53     0.19     0.32     0.25      0.09
  7206    1.50     0.60     0.28     0.37     0.28      0.13
  7207    1.84     0.73     0.42     0.46     0.34      0.19
  7208    1.99     0.87     0.50     0.50     0.40      0.23
  7209    2.38     0.98     0.62     0.59     0.45      0.29
  7210    2.73     1.10     0.84     0.68     0.51      0.39
  7211    2.99     1.25     0.87     0.75     0.58      0.40
  7212    3.61     1.42     0.95     0.90     0.66      0.44
  7213    4.14     1.52     1.02     1.03     0.71      0.47
  7214    4.58     1.60     1.35     1.14     0.74      0.62
  7215    5.02     1.75     1.40     1.25     0.81      0.65
```

```
DATA FOR RUN NUMBER 73.  TOTAL NUMBER OF RUNS IS 23.

WAVE PERIOD IS  2.170SECONDS.    WATER DEPTH IS 14.0INCHES.

DIMENSIONLESS PARAMETERS ARE   NON-DIMENSIONAL PERIOD(=GT**2/H) IS129.844
                                        DEPTH TO RADIUS RATIO(=H/A) IS 7.0000



SFGMENT    WAVE      FORCE     FORCE      H/2A     F. COEF    F. COEF
NUMBER     HEIGHT     (X)        (Y)               (X)          (Y)

  7301      0.43      0.21      0.03      0.11      0.10      0.01
  7302      0.71      0.36      0.05      0.18      0.17      0.02
  7303      1.00      0.45      0.09      0.25      0.21      0.04
  7304      1.25      0.62      0.14      0.31      0.29      0.06
  7305      1.57      0.80      0.22      0.39      0.37      0.10
  7306      1.90      0.89      0.26      0.47      0.41      0.12
  7307      2.16      1.02      0.40      0.54      0.47      0.19
  7308      2.34      1.13      0.50      0.58      0.52      0.23
  7309      2.90      1.28      0.69      0.73      0.59      0.32
  7310      3.30      1.41      0.74      0.82      0.65      0.34
  7311      3.64      1.53      0.92      0.91      0.71      0.42
  7312      4.06      1.95      1.01      1.01      0.90      0.47
  7313      4.50      2.11      1.09      1.13      0.98      0.50
  7314      4.80      2.53      1.43      1.20      1.17      0.66
  7315      5.16      2.56      1.57      1.29      1.18      0.72
```

```
DATA FOR RUN NUMBER 74.  TOTAL NUMBER OF RUNS IS 23.

WAVE PERIOD IS  2.022SECONDS.    WATER DEPTH IS 14.0INCHES.

DIMENSIONLESS PARAMETERS ARE   NON-DIMENSIONAL PERIOD(=GT**2/H) IS112.793
                                        DEPTH TO RADIUS RATIO(=H/A) IS 7.0000



SEGMENT   WAVE     FORCE    FORCE    H/2A    F. COEF   F. COEF
NUMBER    HEIGHT    (X)       (Y)            (X)         (Y)

  7401     0.52     0.22     0.04    0.13     0.10     0.02
  7402     0.90     0.38     0.05    0.23     0.18     0.02
  7403     1.19     0.55     0.11    0.30     0.25     0.05
  7404     1.54     0.69     0.19    0.38     0.32     0.09
  7405     1.85     0.82     0.31    0.46     0.38     0.14
  7406     2.30     0.93     0.35    0.57     0.43     0.16
  7407     2.74     1.07     0.40    0.69     0.50     0.19
  7408     3.16     1.33     0.54    0.79     0.62     0.25
  7409     3.26     1.50     0.70    0.81     0.69     0.33
  7410     3.60     1.81     0.84    0.90     0.84     0.39
  7411     4.26     2.00     0.97    1.06     0.93     0.45
  7412     4.96     2.22     1.21    1.24     1.03     0.56
  7413     5.20     2.39     1.30    1.30     1.11     0.60
  7414     5.76     2.56     1.51    1.44     1.18     0.70
```

```
DATA FOR RUN NUMBER 75.  TOTAL NUMBER OF RUNS IS 23.

WAVE PFRIOD IS  2.447SECONDS.    WATER DEPTH IS 14.0INCHES.

DIMENSIONLESS PARAMETERS ARE   NON-DIMENSIONAL PERIOD(=GT**2/H) IS165.177
                                         DEPTH TO RADIUS RATIO(=H/A) IS 7.0000



SFGMENT    WAVE      FORCE     FORCE      H/2A     F. COEF    F. COEF
NUMBER     HEIGHT    (X)         (Y)               (X)          (Y)

  7501     0.38      0.16      0.01      0.09      0.08       0.01
  7502     0.62      0.30      0.03      0.15      0.14       0.01
  7503     0.85      0.44      0.07      0.21      0.20       0.03
  7504     1.04      0.55      0.13      0.26      0.25       0.06
  7505     1.42      0.74      0.19      0.36      0.34       0.09
  7506     1.61      0.76      0.34      0.40      0.35       0.16
  7507     1.83      0.85      0.45      0.46      0.39       0.21
  7508     2.20      1.02      0.46      0.55      0.47       0.21
  7509     2.50      1.19      0.86      0.62      0.55       0.40
  7510     3.10      1.21      0.86      0.78      0.56       0.40
  7511     3.40      1.36      0.97      0.85      0.63       0.45
  7512     3.80      1.47      1.04      0.95      0.68       0.48
  7513     4.20      1.58      1.29      1.05      0.73       0.59
  7514     4.60      1.95      1.43      1.15      0.90       0.66
  7515     5.00      2.00      1.51      1.25      0.93       0.70
```

```
DATA FOR RUN NUMBER 91.  TOTAL NUMBER OF RUNS IS 23.

WAVE PERIOD IS  2.665SECONDS.    WATER DEPTH IS 18.0INCHES.

DIMENSIONLESS PARAMETERS ARE    NON-DIMENSIONAL PERIOD(=GT**2/H) IS152.319
                                     DEPTH TO RADIUS RATIO(=H/A) IS 9.0000



SEGMENT    WAVE      FORCE     FORCE      H/2A      F. COEF    F. COEF
NUMBER     HEIGHT     (X)       (Y)                 (X)         (Y)


  9101     0.47      0.11      0.01      0.12       0.05       0.00
  9102     0.66      0.18      0.01      0.17       0.08       0.01
  9103     1.04      0.27      0.03      0.26       0.13       0.01
  9104     1.41      0.38      0.07      0.35       0.18       0.03
  9105     1.71      0.51      0.11      0.43       0.24       0.05
  9106     1.84      0.61      0.15      0.46       0.28       0.07
  9107     2.30      0.71      0.25      0.57       0.33       0.11
  9108     2.58      0.84      0.38      0.64       0.39       0.18
  9109     3.02      0.88      0.50      0.76       0.41       0.23
  9110     3.20      0.93      0.68      0.80       0.43       0.31
  9111     3.50      1.02      0.85      0.87       0.47       0.39
  9112     3.70      1.11      0.86      0.92       0.51       0.40
  9113     4.20      1.23      1.00      1.05       0.57       0.46
  9114     4.56      1.30      1.27      1.14       0.60       0.59
  9115     5.20      1.37      1.41      1.30       0.63       0.65
```

```
DATA FOR RUN NUMBER 92.  TOTAL NUMBER OF RUNS IS 23.

WAVE PERIOD IS  2.342SECONDS.    WATER DEPTH IS 18.0INCHES.

DIMENSIONLESS PARAMETERS ARE   NON-DIMENSIONAL PERIOD(=GT**2/H) IS117.684
                                   DEPTH TO RADIUS RATIO(=H/A) IS 9.0000


SEGMENT   WAVE     FORCE    FORCE     H/2A     F. COEF   F. COEF
NUMBER    HEIGHT    (X)      (Y)               (X)        (Y)

  9201     0.58     0.15     0.01     0.14      0.07      0.00
  9202     0.95     0.28     0.05     0.24      0.13      0.02
  9203     1.19     0.40     0.11     0.30      0.18      0.05
  9204     1.63     0.52     0.18     0.41      0.24      0.08
  9205     1.99     0.65     0.25     0.50      0.30      0.11
  9206     2.75     0.66     0.33     0.69      0.31      0.15
  9207     2.84     0.93     0.38     0.71      0.43      0.17
  9208     2.94     1.06     0.51     0.74      0.49      0.24
  9209     3.26     1.03     0.72     0.81      0.48      0.33
  9210     3.60     1.16     0.89     0.90      0.54      0.41
  9211     4.00     1.25     0.91     1.00      0.58      0.42
  9212     4.38     1.36     0.97     1.09      0.63      0.45
  9213     4.70     1.67     1.05     1.17      0.77      0.49
  9214     5.14     1.86     1.31     1.28      0.86      0.60
  9215     5.50     1.95     1.43     1.38      0.90      0.66
```

```
DATA FOR RUN NUMBER 93.  TOTAL NUMBER OF RUNS IS 23.

WAVE PERIOD IS  2.430SECONDS.     WATER DEPTH IS 18.0INCHES.

DIMENSIONLESS PARAMETERS ARE   NON-DIMENSIONAL PERIOD(=GT**2/H) IS126.640
                                         DEPTH TO RADIUS RATIO(=H/A) IS 9.0000



SEGMENT   WAVE     FORCE    FORCE     H/2A     F. COEF    F. COEF
NUMBER    HEIGHT    (X)       (Y)               (X)         (Y)

  9301     0.50     0.14     0.0      0.12      0.06       0.0
  9302     0.83     0.19     0.03     0.21      0.09       0.01
  9303     1.24     0.27     0.08     0.31      0.13       0.04
  9304     1.58     0.38     0.11     0.39      0.18       0.05
  9305     1.91     0.55     0.13     0.48      0.25       0.06
  9306     2.28     0.66     0.26     0.57      0.31       0.12
  9307     2.90     0.77     0.32     0.73      0.36       0.15
  9308     3.00     0.88     0.39     0.75      0.41       0.18
  9309     3.60     0.93     0.46     0.90      0.43       0.21
  9310     3.90     1.00     0.63     0.98      0.46       0.29
  9311     4.30     1.05     0.73     1.07      0.49       0.34
  9312     4.80     1.26     0.94     1.20      0.59       0.44
  9313     5.00     1.40     1.19     1.25      0.65       0.55
  9314     5.60     1.55     1.35     1.40      0.72       0.62
  9315     6.00     1.69     1.40     1.50      0.78       0.65
```

```
DATA FOR RUN NUMBER 94.  TOTAL NUMBER OF RUNS IS 23.

WAVE PERIOD IS  2.097SECONDS.     WATER DEPTH IS 18.0INCHES.

DIMENSIONLESS PARAMETERS ARE   NON-DIMENSIONAL PERIOD(=GT**2/H) IS 94.355
                                        DEPTH TO RADIUS RATIO(=H/A) IS 9.0000



SEGMENT    WAVE      FORCE     FORCE      H/2A     F. COEF    F. COEF
NUMBER     HEIGHT     (X)        (Y)              (X)          (Y)


  9401      0.58      0.16      0.02      0.15      0.08       0.01
  9402      0.91      0.33      0.04      0.23      0.15       0.02
  9403      1.29      0.47      0.13      0.32      0.22       0.06
  9404      1.66      0.60      0.18      0.41      0.28       0.08
  9405      1.91      0.63      0.31      0.48      0.29       0.14
  9406      2.32      0.83      0.40      0.58      0.39       0.19
  9407      2.80      0.94      0.45      0.70      0.44       0.21
  9408      3.30      1.11      0.51      0.82      0.51       0.24
  9409      3.64      1.22      0.63      0.91      0.56       0.29
  9410      3.90      1.40      0.70      0.98      0.65       0.32
  9411      4.26      1.63      0.81      1.06      0.75       0.37
  9412      5.00      1.74      0.92      1.25      0.81       0.42
  9413      5.50      1.97      1.27      1.38      0.91       0.59
  9414      6.00      2.02      1.34      1.50      0.94       0.62
  9415      6.70      2.25      1.43      1.67      1.04       0.66
```

```
DATA FOR RUN NUMBER 95.  TOTAL NUMBER OF RUNS IS 23.

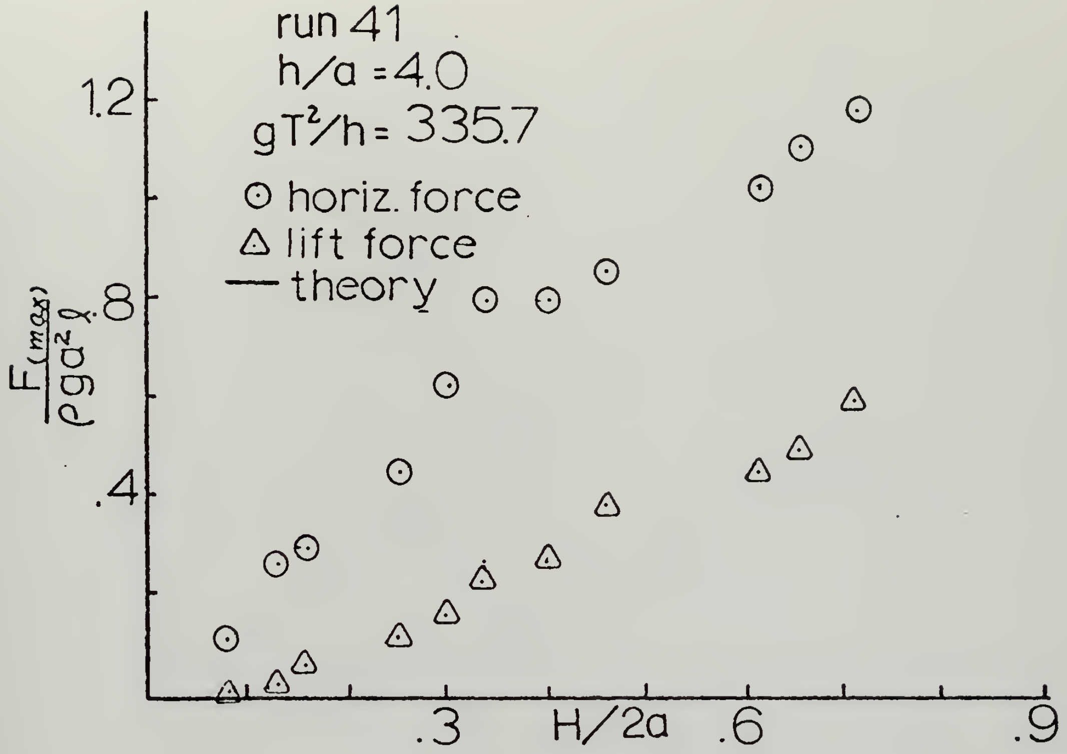
WAVE PERIOD IS  1.957SECONDS.     WATER DEPTH IS 18.0INCHES.

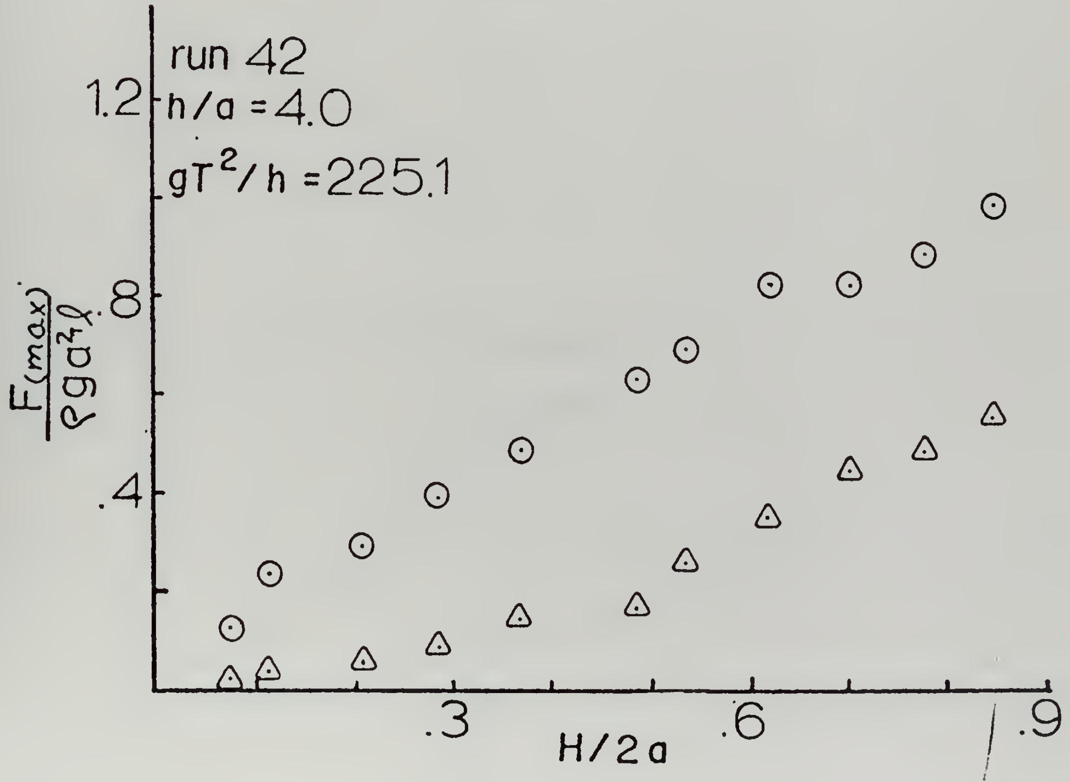
DIMENSIONLESS PARAMETERS ARE   NON-DIMENSIONAL PERIOD(=GT**2/H) IS 82.179
                                     DEPTH TO RADIUS RATIO(=H/A) IS 9.0000

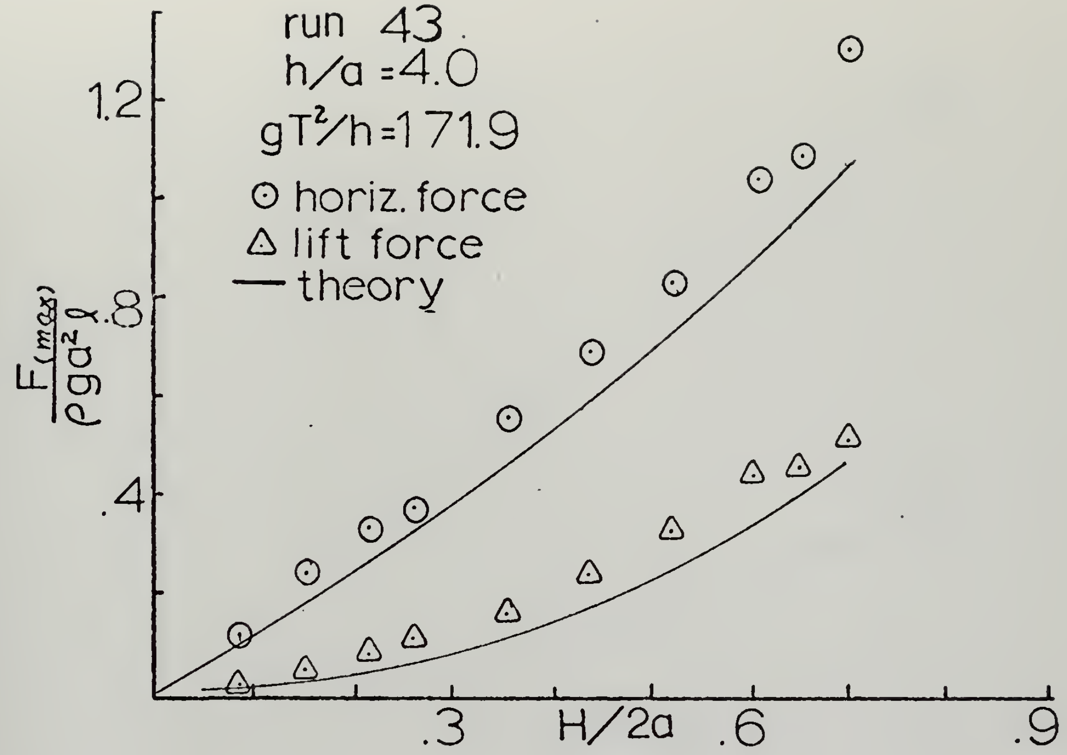


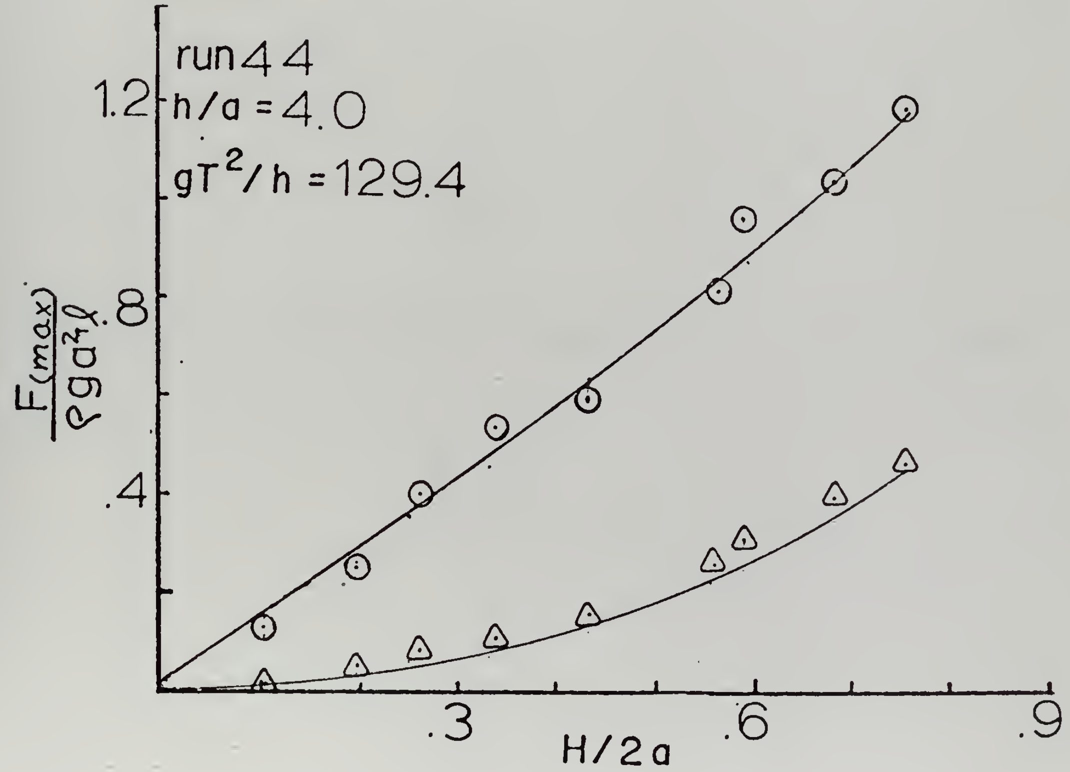
SEGMENT   WAVE      FORCE     FORCE      H/2A     F. COEF    F. COEF
NUMBER    HEIGHT     (X)        (Y)               (X)         (Y)

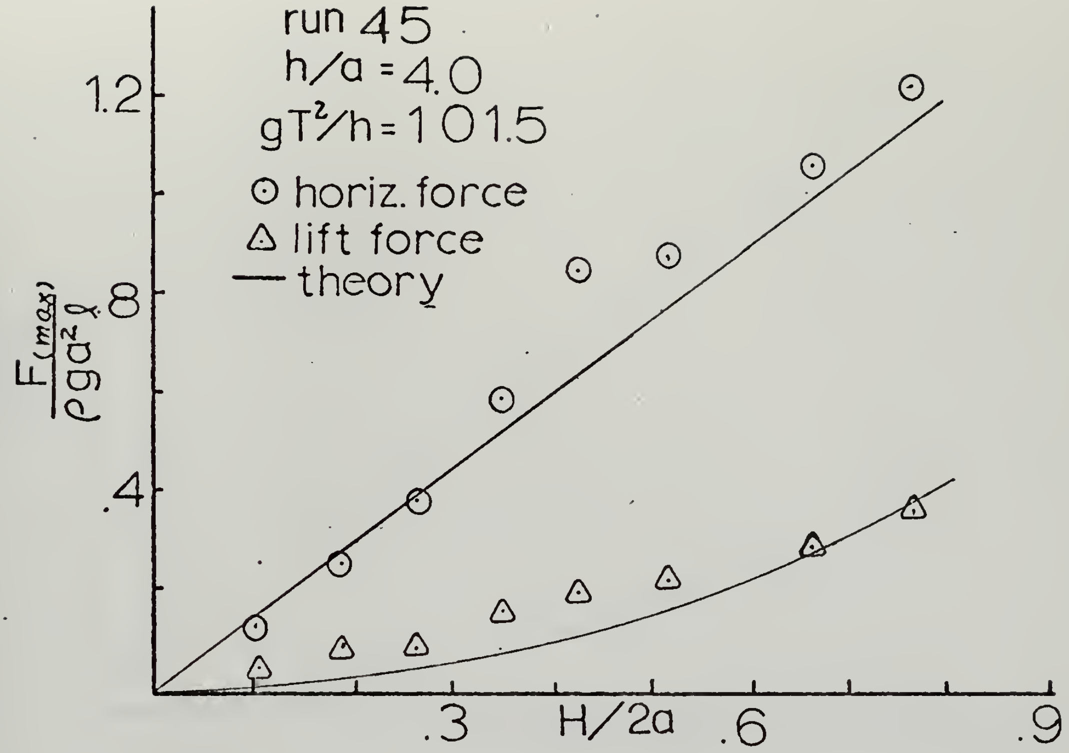
  9501     0.58      0.17      0.01      0.15      0.08       0.01
  9502     1.04      0.34      0.04      0.26      0.16       0.02
  9503     1.45      0.47      0.09      0.36      0.22       0.04
  9504     1.74      0.66      0.15      0.44      0.31       0.07
  9505     1.91      0.69      0.27      0.48      0.32       0.12
  9506     3.00      0.83      0.35      0.75      0.39       0.16
  9507     3.40      0.99      0.39      0.85      0.46       0.18
  9508     3.80      1.11      0.54      0.95      0.51       0.25
  9509     4.20      1.32      0.70      1.05      0.61       0.32
  9510     4.60      1.44      0.86      1.15      0.67       0.40
  9511     5.40      1.66      0.93      1.35      0.77       0.43
  9512     5.70      2.11      0.97      1.42      0.98       0.45
  9513     6.20      2.39      1.08      1.55      1.11       0.50
  9514     6.80      2.50      1.40      1.70      1.16       0.65
  9515     7.40      2.53      1.73      1.85      1.17       0.80
```

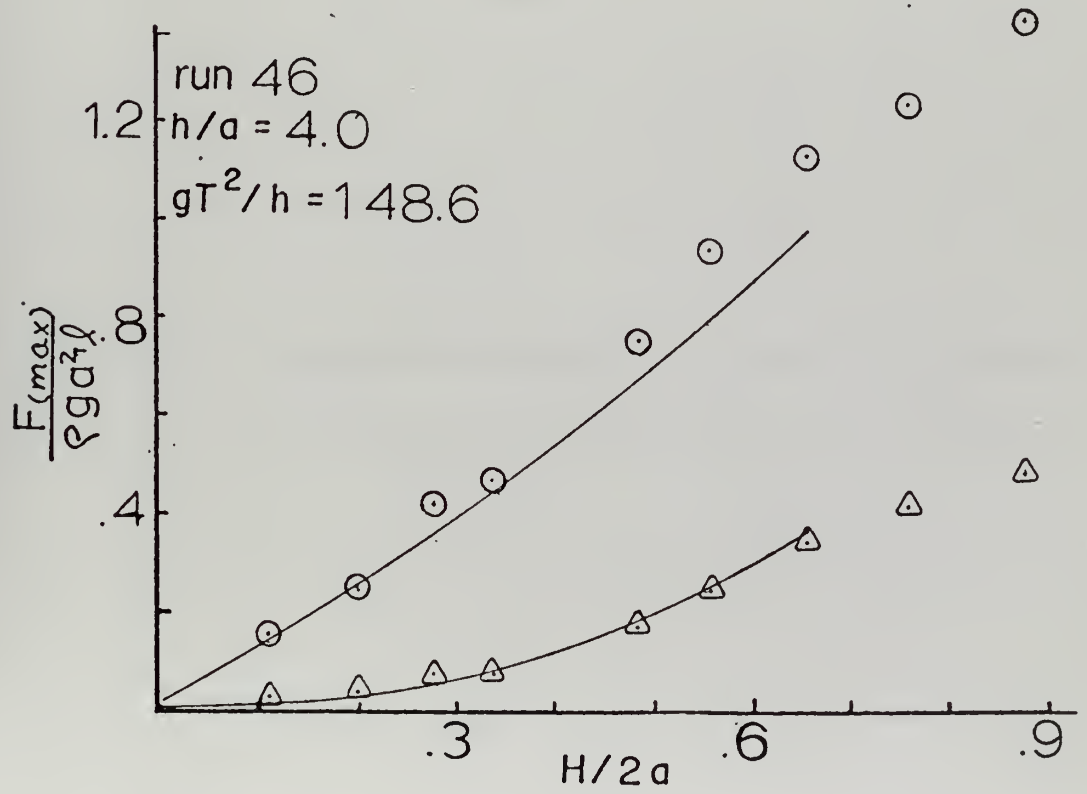

run 41
h/a = 4.0
gT²/h = 335.7
⊙ horiz. force
△ lift force
— theory
1.2
.8
.4
$\frac{F_{(max)}}{\rho g a^2 \ell}$
.3
H/2a
.6
.9

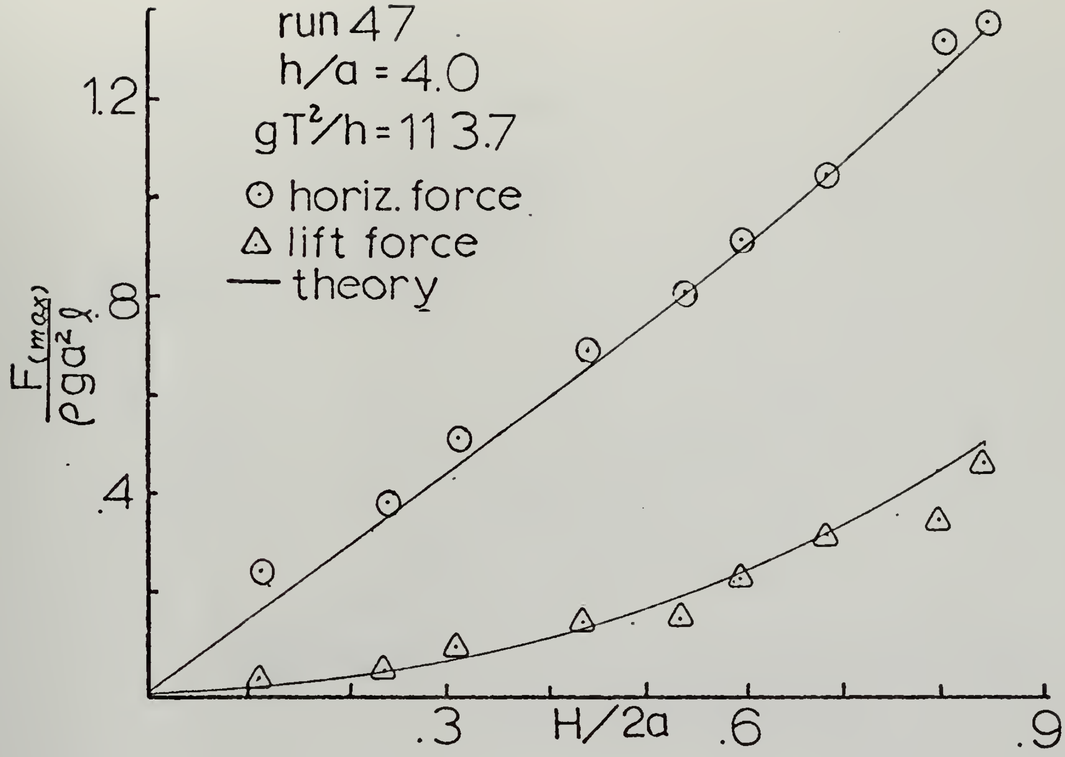

run 42
h/a = 4.0
gT²/h = 225.1
1.2
.8
.4
$\frac{F_{(max)}}{\rho g a^2 \ell}$
.3
.6
.9
H/2a

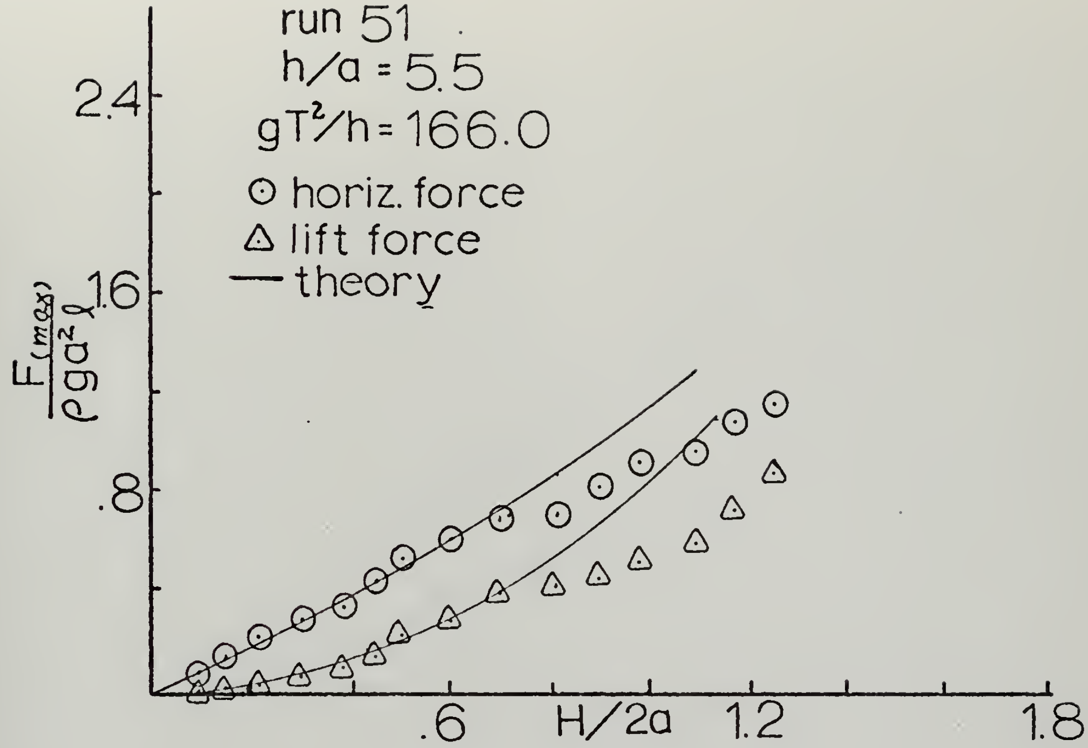

run 43
h/a = 4.0
gT²/h = 171.9
⊙ horiz. force
△ lift force
— theory
1.2
.8
.4
$\frac{F_{(max)}}{\rho g a^2 \ell}$
.3
H/2a
.6
.9

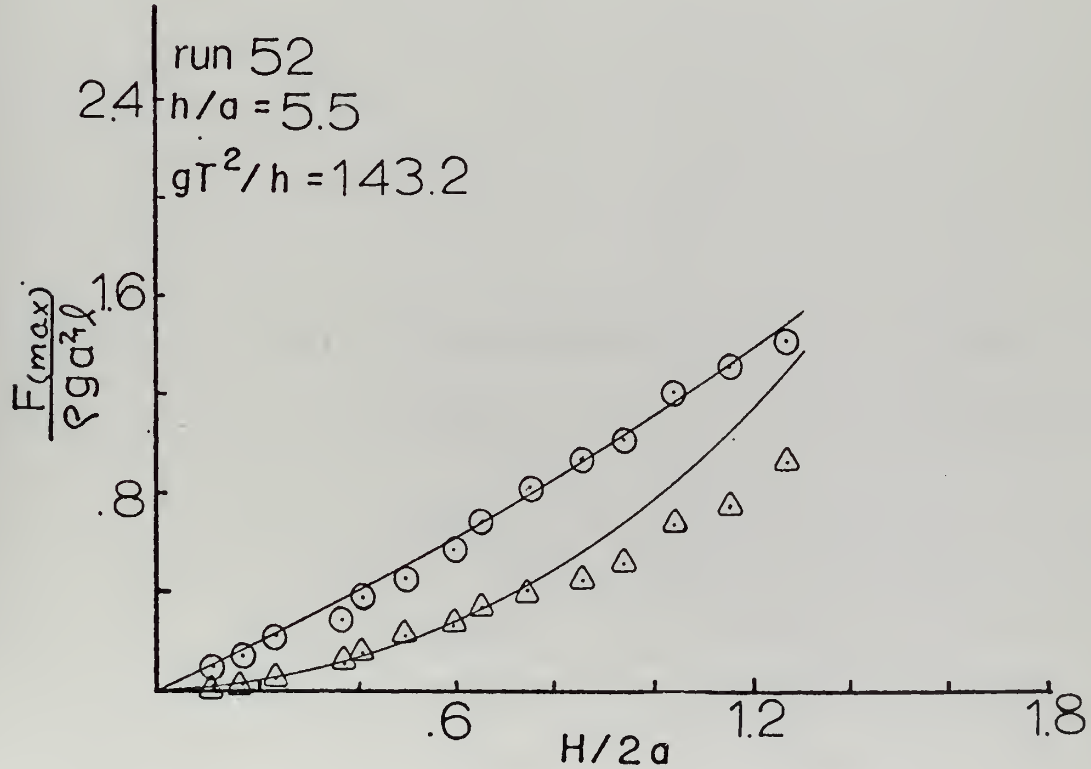

run 44
h/a = 4.0
gT²/h = 129.4
1.2
.8
.4
$\frac{F_{(max)}}{\rho g a^2 \ell}$
.3
.6
.9
H/2a

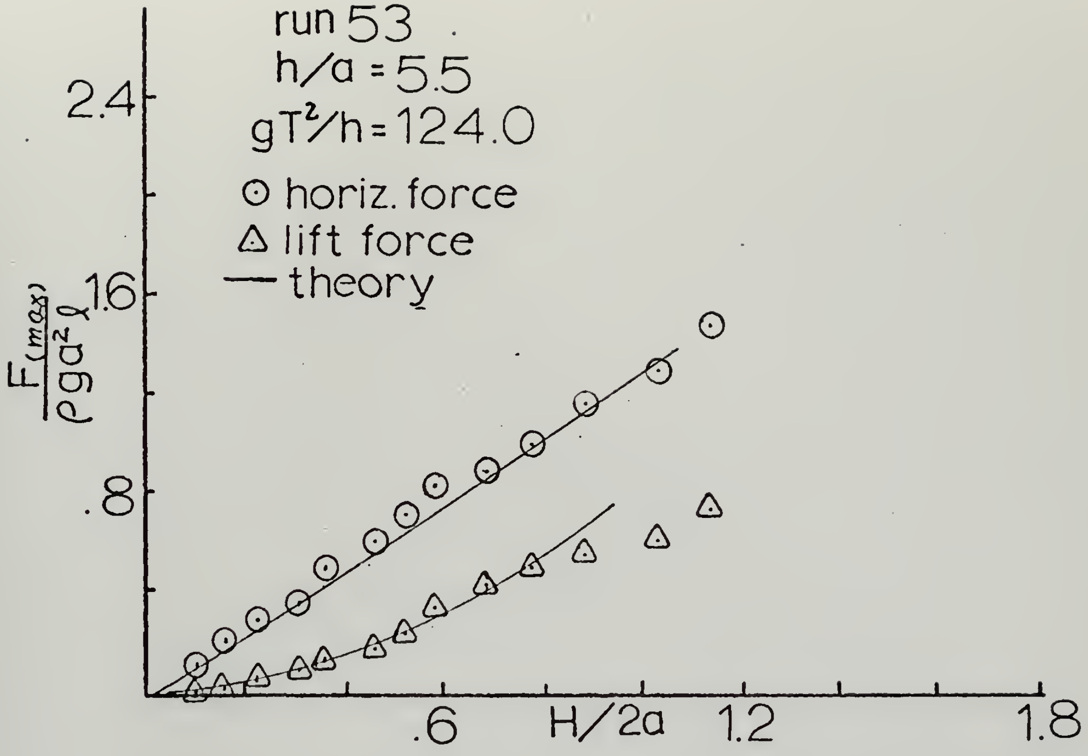

run 45
h/a = 4.0
$gT^2/h = 101.5$
⊙ horiz. force
△ lift force
— theory
1.2
.8
.4
$\frac{F_{(max)}}{\rho g a^2 \ell}$
.3
H/2a
.6
.9

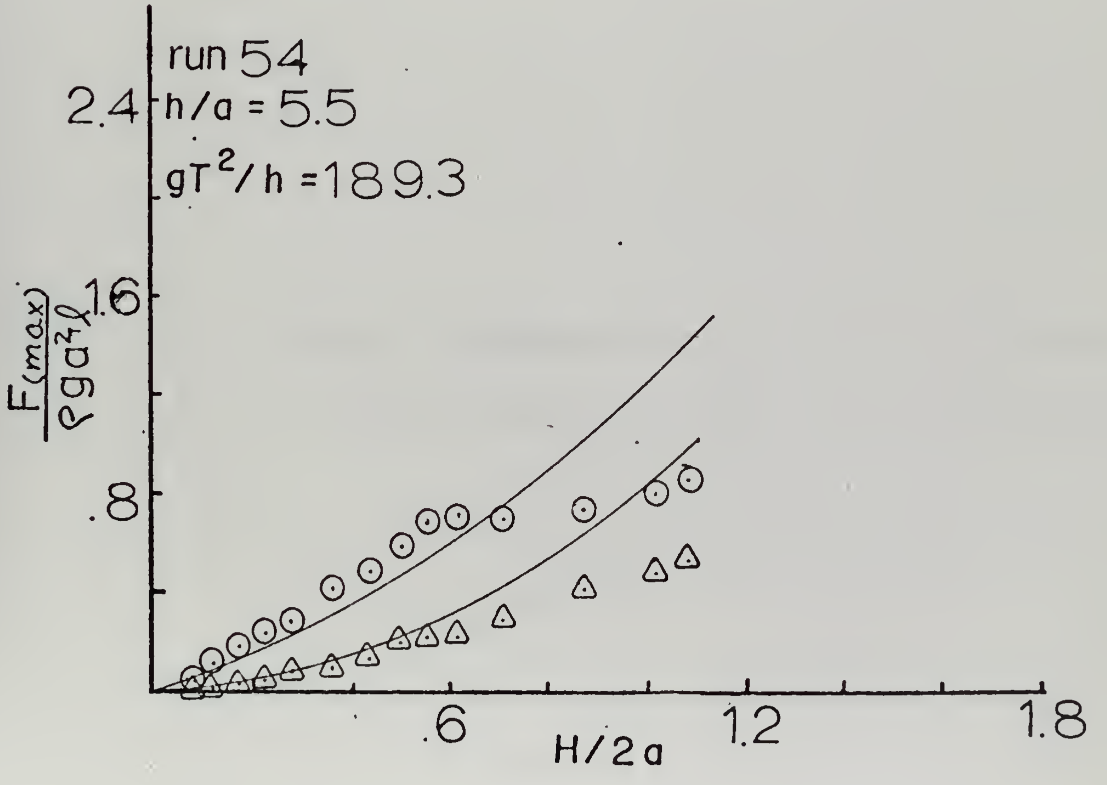

run 46
h/a = 4.0
$gT^2/h = 148.6$
1.2
.8
.4
$\frac{F_{(max)}}{\rho g a^2 \ell}$
.3
.6
.9
H/2a

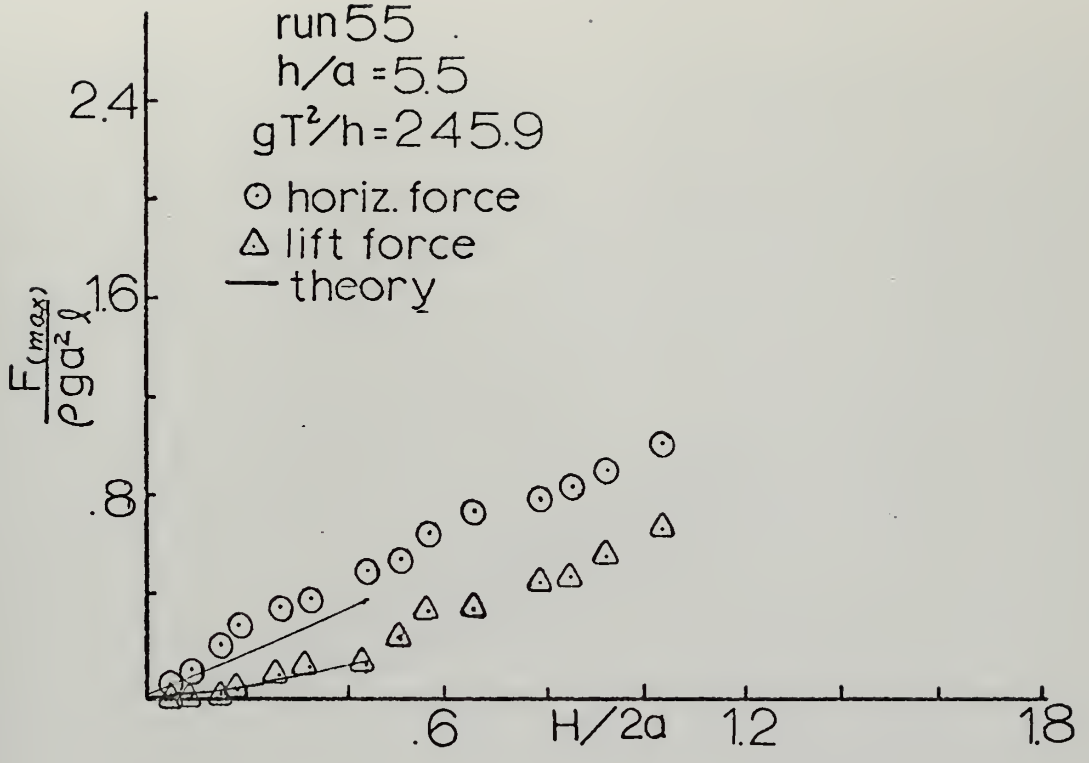

run 47
h/a = 4.0
gT²/h = 113.7
⊙ horiz. force
△ lift force
— theory
1.2
.8
.4
F(max) / ρga²ℓ
.3
H/2a
.6
.9

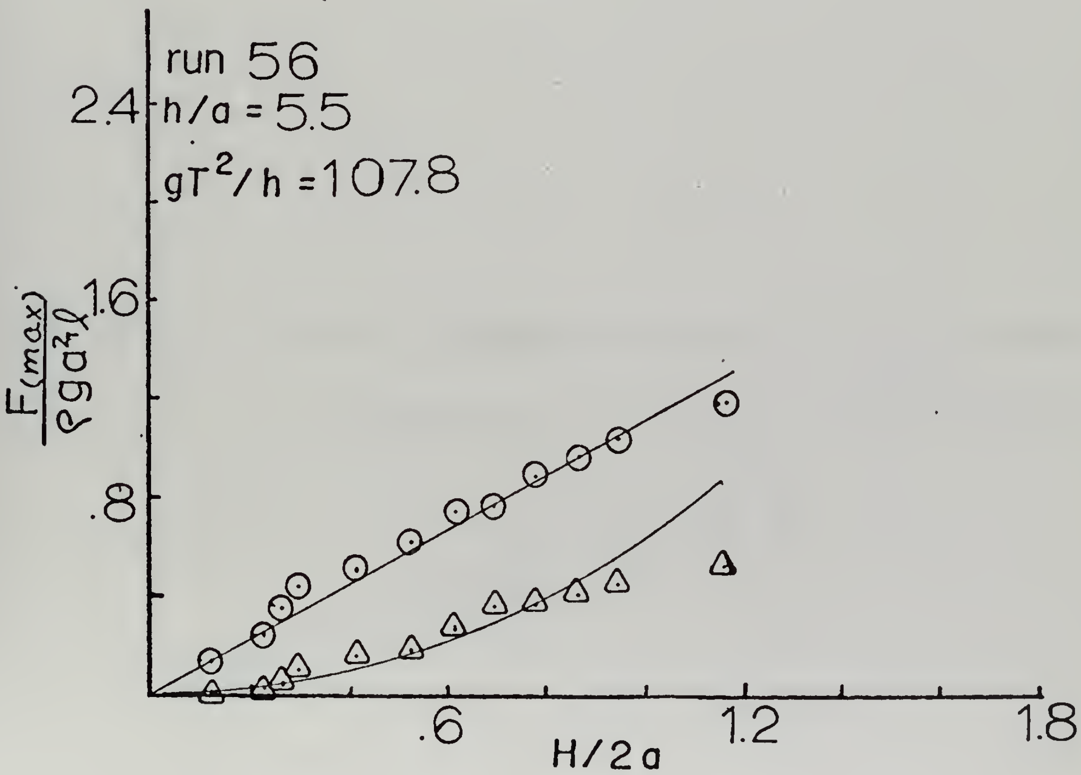

run 51
h/a = 5.5
gT²/h = 166.0
horiz. force
lift force
theory
2.4
1.6
.8
$\frac{F_{(max)}}{\rho g a^2 \ell}$
.6
H/2a
1.2
1.8

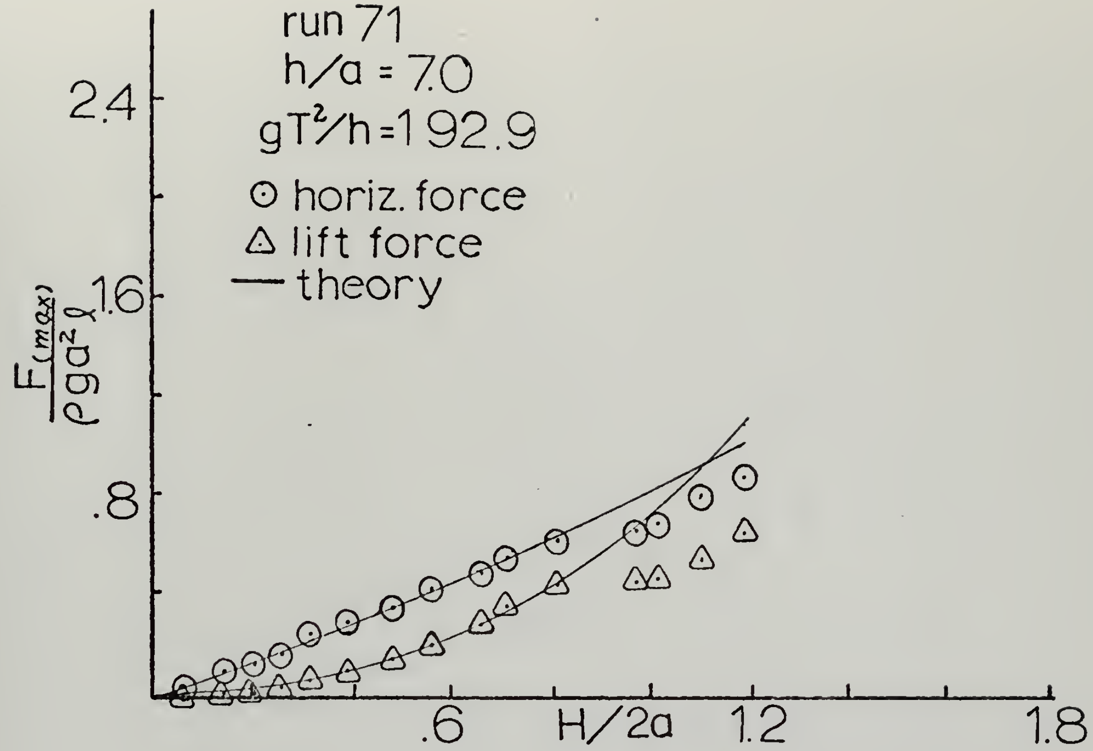

run 52
h/a = 5.5
gT²/h = 143.2
2.4
1.6
.8
$\frac{F_{(max)}}{\rho g a^2 \ell}$
.6
1.2
1.8
H/2a

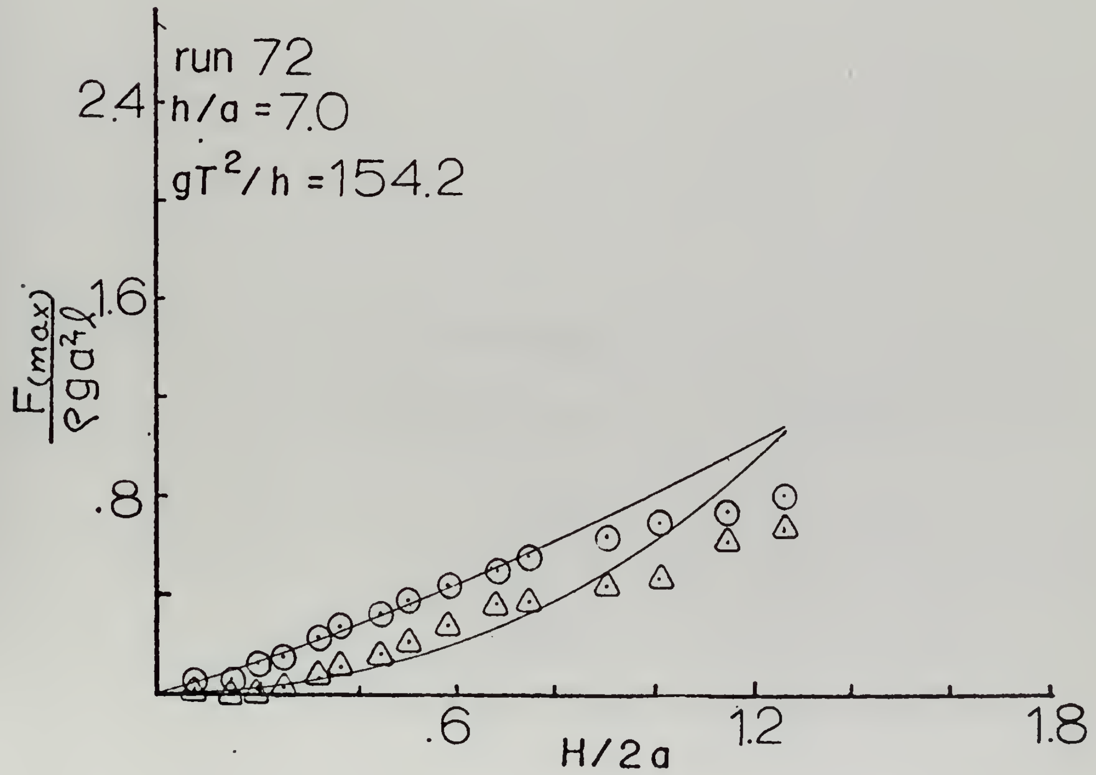

run 53
h/a = 5.5
gT²/h = 124.0
⊙ horiz. force
△ lift force
— theory
2.4
1.6
.8
$\frac{F_{(max)}}{\rho g a^2 \ell}$
.6
H/2a
1.2
1.8

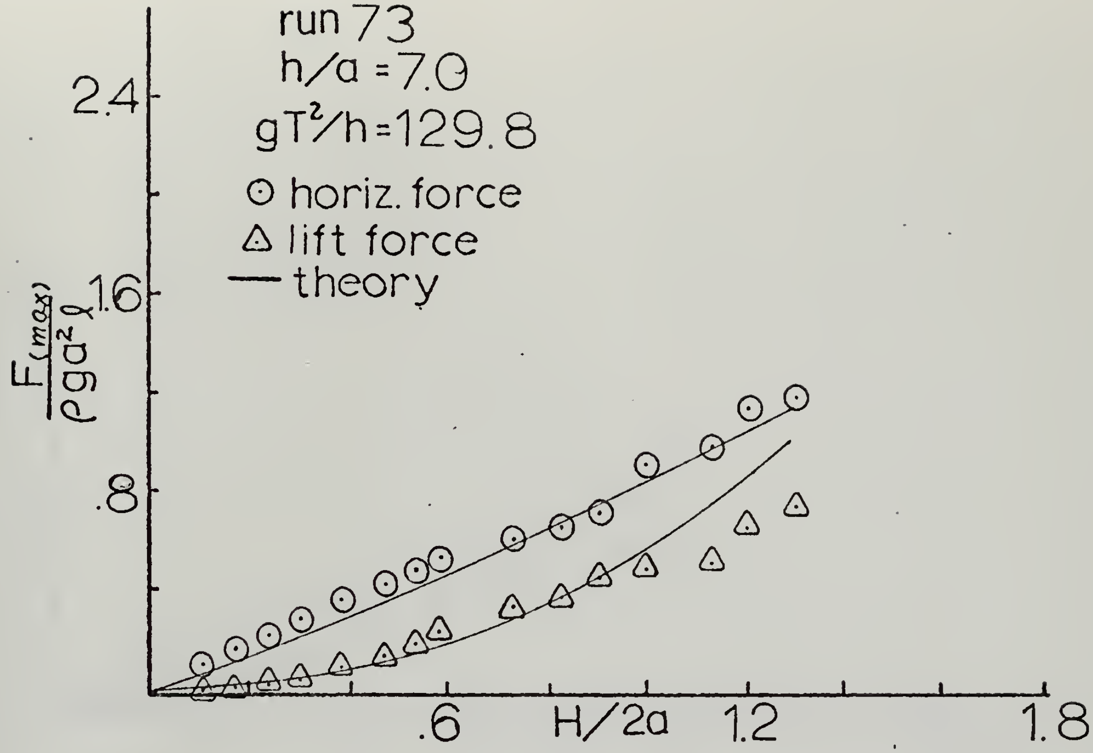

run 54
h/a = 5.5
gT²/h = 189.3
2.4
1.6
.8
$\frac{F_{(max)}}{\rho g a^2 \ell}$
.6
1.2
1.8
H/2a

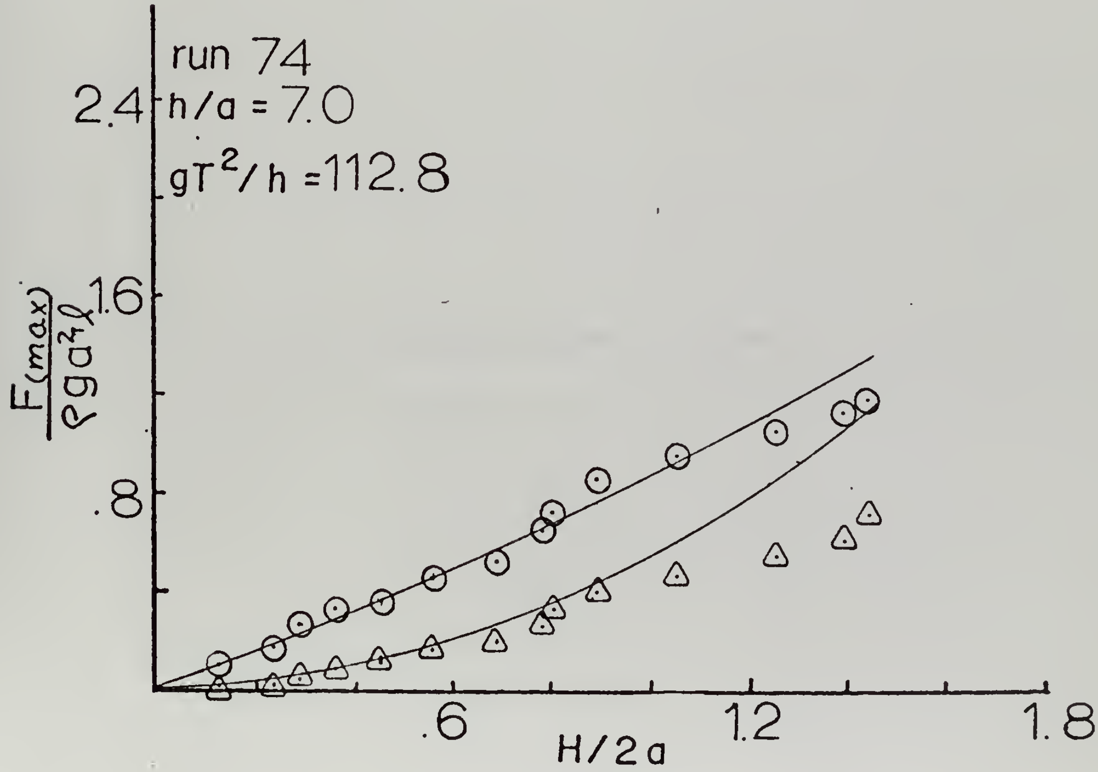

run55
h/a =5.5
gT²/h=245.9
⊙ horiz. force
△ lift force
— theory
2.4
1.6
.8
$\frac{F_{(max)}}{\rho g a^2 \ell}$
.6
H/2a
1.2
1.8

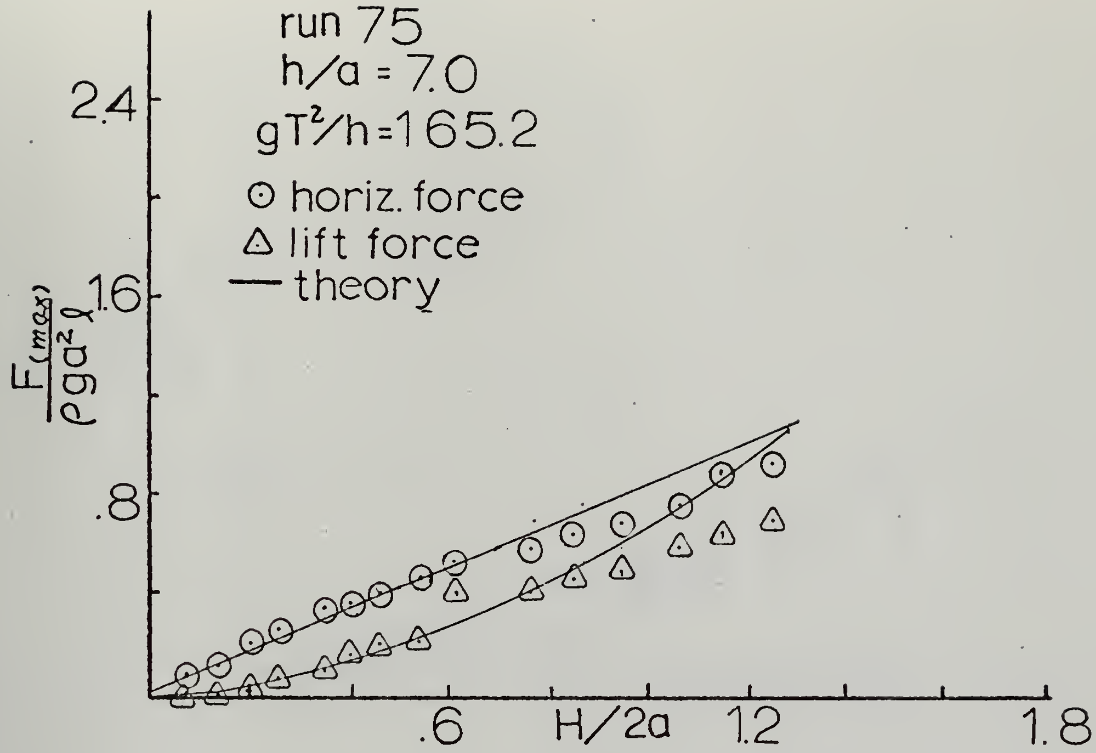

run 56
h/a = 5.5
gT²/h =107.8
2.4
1.6
.8
$\frac{F_{(max)}}{\rho g a^2 \ell}$
.6
1.2
1.8
H/2a

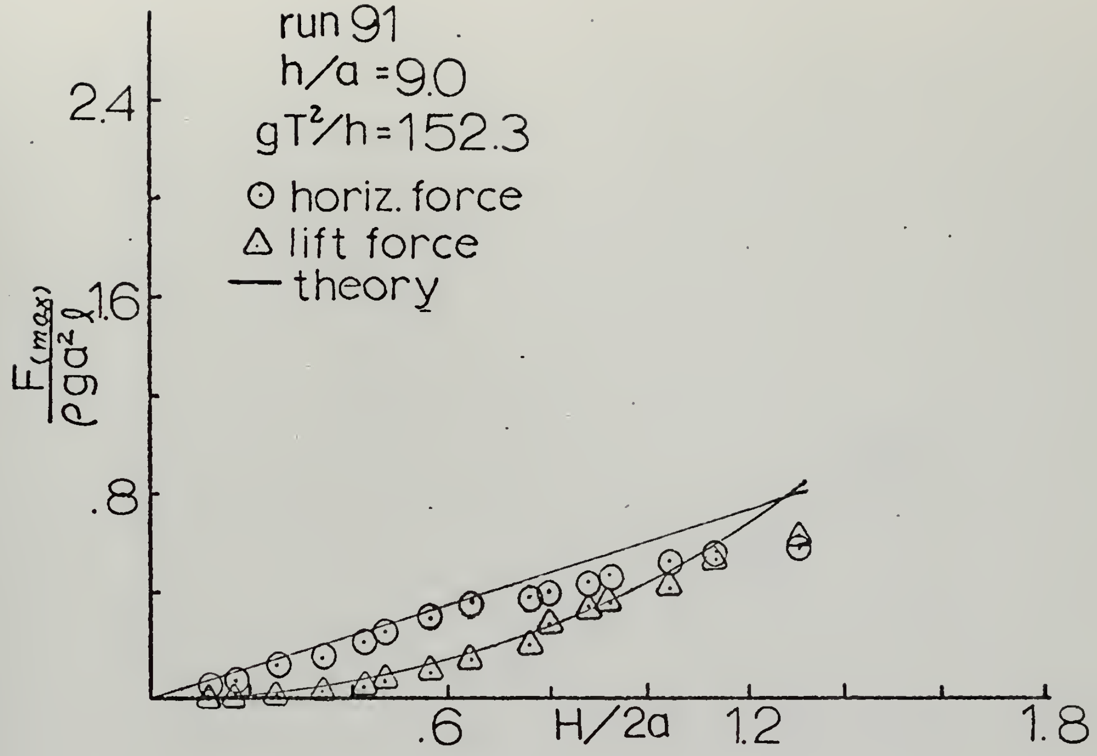

run 71
h/a = 7.0
gT²/h = 192.9
⊙ horiz. force
△ lift force
— theory
$\frac{F_{(max)}}{\rho g a^2 \ell}$
2.4
1.6
.8
.6
H/2a
1.2
1.8

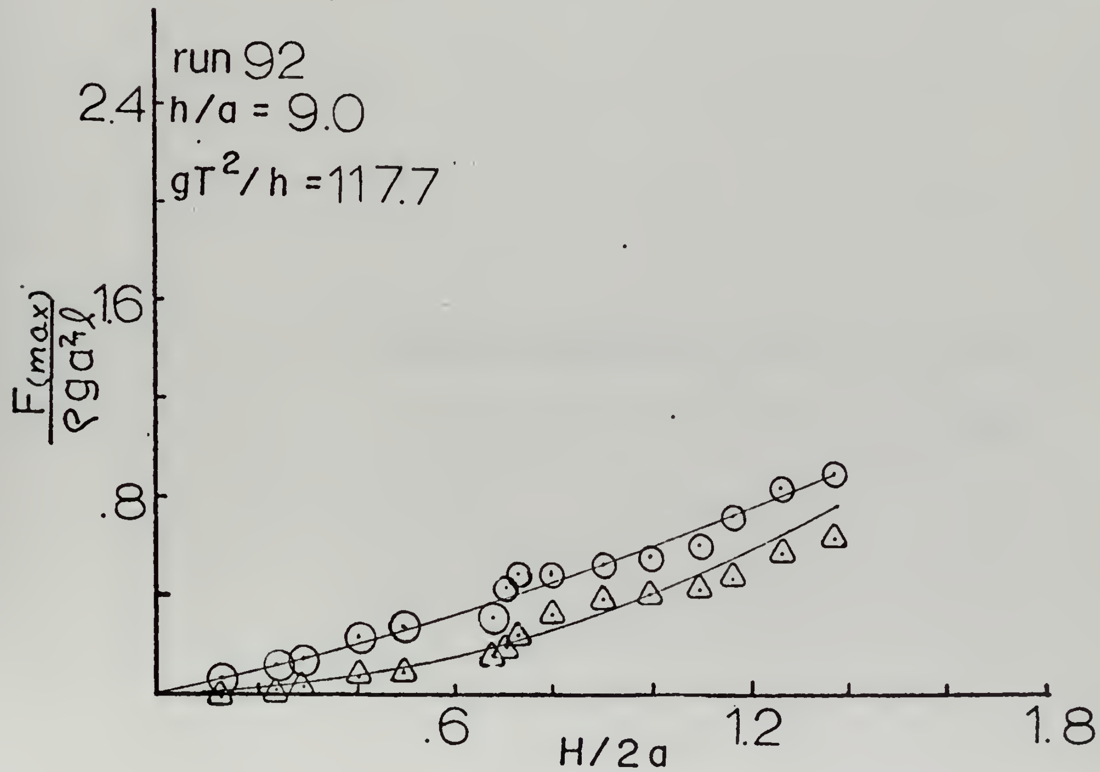

run 72
h/a = 7.0
gT²/h = 154.2
$\frac{F_{(max)}}{\rho g a^2 \ell}$
2.4
1.6
.8
.6
1.2
1.8
H/2a

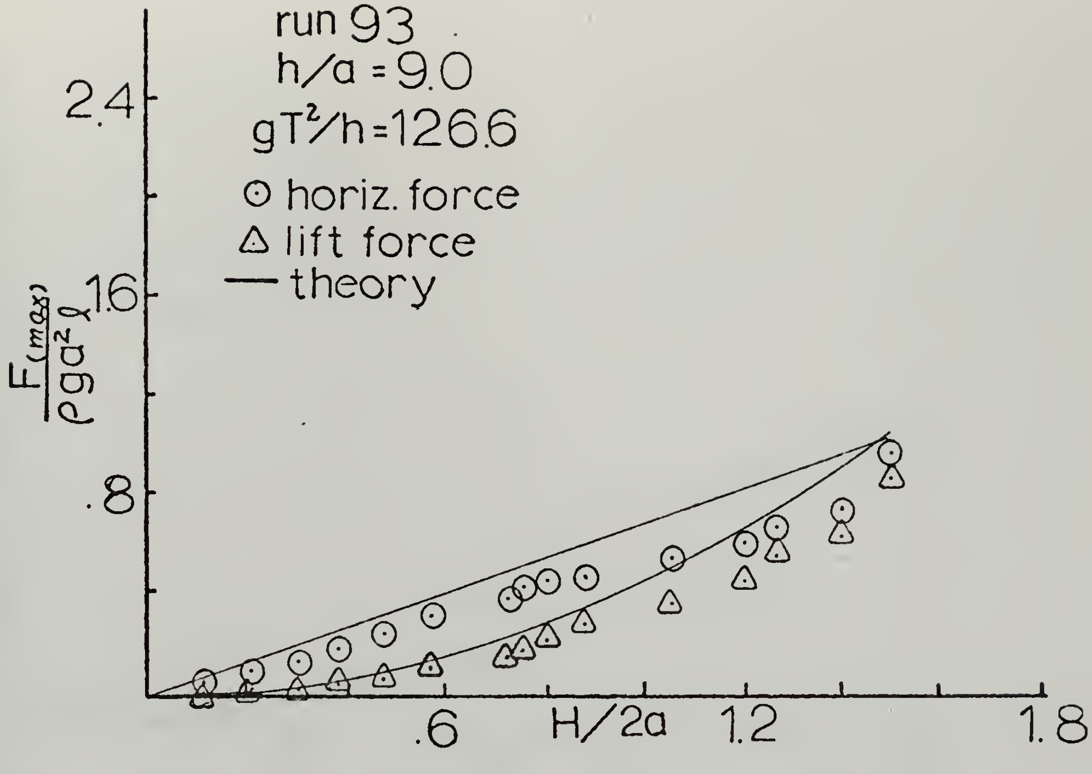

run 73
h/a = 7.0
gT²/h = 129.8
⊙ horiz. force
△ lift force
— theory
$\frac{F_{(max)}}{\rho g a^2 \ell}$
2.4
1.6
.8
.6
H/2a
1.2
1.8

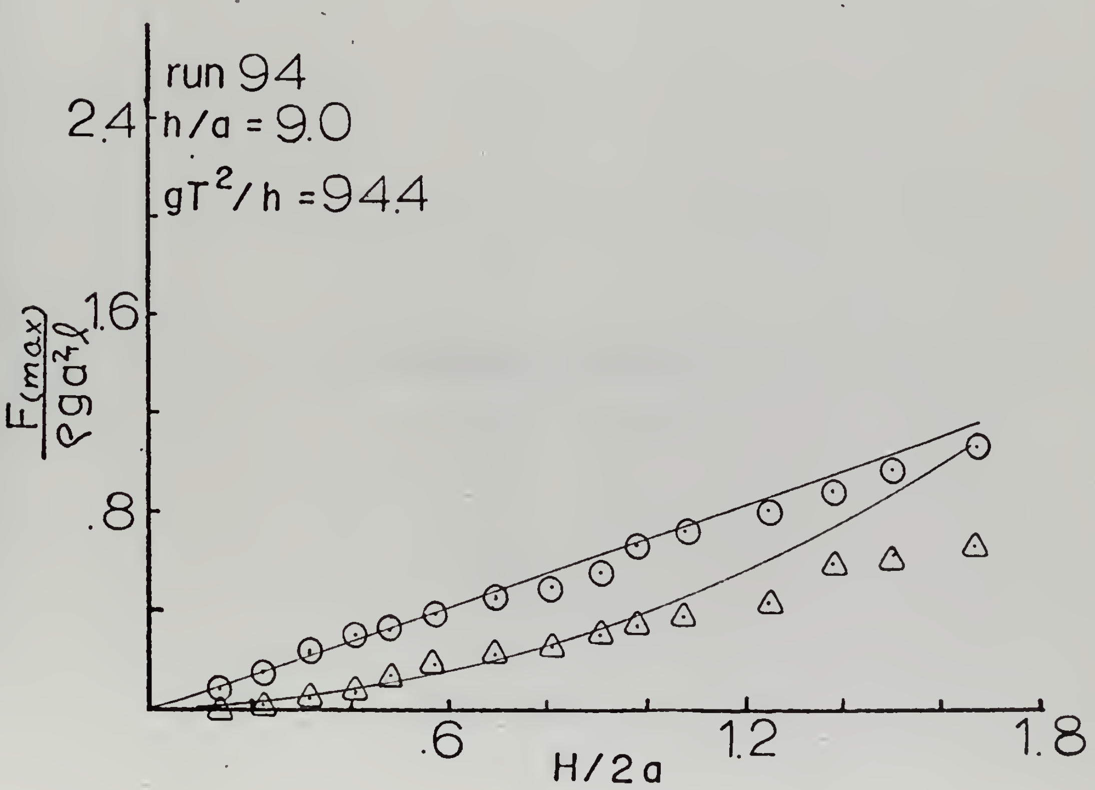

run 74
h/a = 7.0
gT²/h = 112.8
$\frac{F_{(max)}}{\rho g a^2 \ell}$
2.4
1.6
.8
.6
1.2
1.8
H/2a

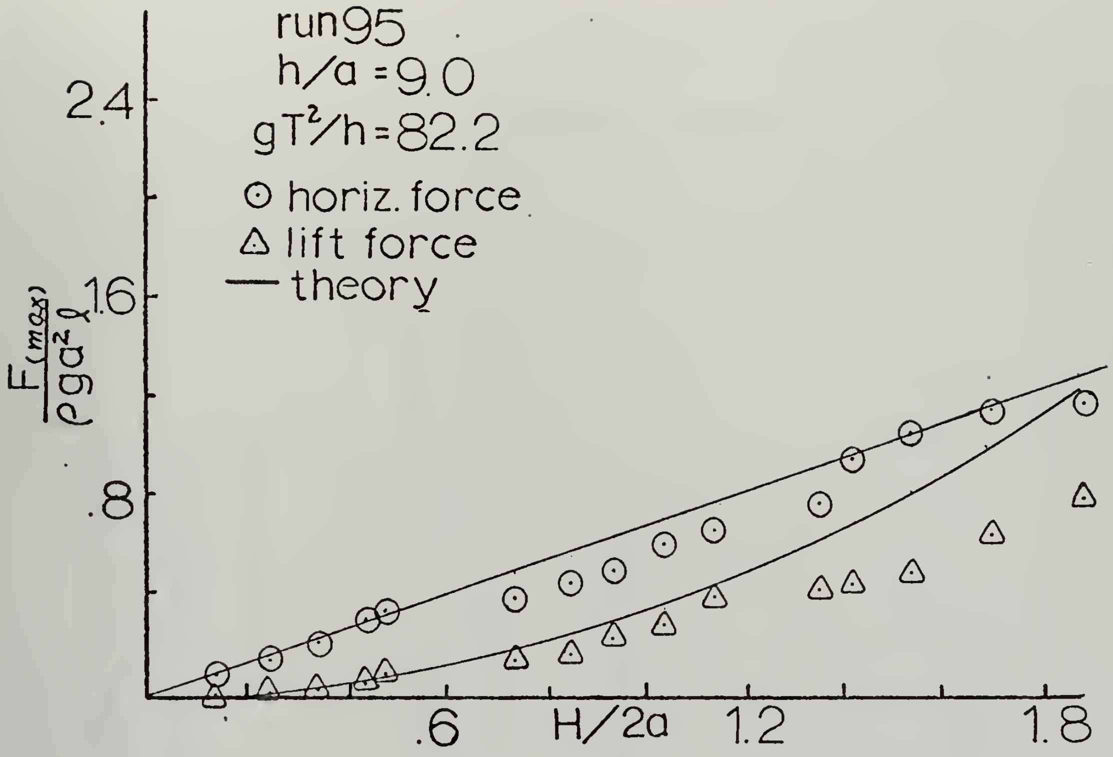

run 75
h/a = 7.0
gT²/h = 165.2
⊙ horiz. force
△ lift force
— theory
2.4
1.6
.8
$\frac{F_{(max)}}{\rho g a^2 \ell}$
.6
H/2a
1.2
1.8


run 91
h/a = 9.0
gT²/h = 152.3
⊙ horiz. force
△ lift force
— theory
2.4
1.6
.8
F(max) / ρga²ℓ
.6
H/2a
1.2
1.8


run 92
h/a = 9.0
gT²/h = 117.7
2.4
1.6
.8
F(max) / ρga²ℓ
.6
1.2
1.8
H/2a


run 93
h/a = 9.0
gT²/h = 126.6
⊙ horiz. force
△ lift force
— theory
$\frac{F_{(max)}}{\rho g a^2 \ell}$
2.4
1.6
.8
.6
H/2a
1.2
1.8


run 94
h/a = 9.0
gT²/h = 94.4
$\frac{F_{(max)}}{\rho g a^2 \ell}$
2.4
1.6
.8
.6
1.2
1.8
H/2a


run95
h/a = 9.0
gT²/h = 82.2
⊙ horiz. force
△ lift force
— theory
2.4
1.6
.8
$\frac{F_{(max)}}{\rho g a^2 l}$
.6
H/2a
1.2
1.8

APPENDIX D

COMPUTER PROGRAM

```
          FORCES ON A HORIZONTAL CYLINDER DUE TO NON-LINEAR WAVES

          THESIS TOPIC                    FRED HERMAN GEHRMAN JR.

          NAVAL POSTGRADUATE SCHOOL                  DECENBER,1972



      THIS PROGRAM COMPUTES THE FORCES ON A BOTTOM-MOORED RIGHT CIRCULAR

      CYLINDER DUE TO WAVE MOTION USING STOKE'S FIFTH ORDER GRAVITY WAVE

      THEORY. THE INPUTS TO THE PROGRAM ARE
            TN=NONDIMENSIONAL PERIOD PARAMETER=G*(T**2)/DEPTH
            DN=DEPTH/CYLINDER RADIUS
            HN=WAVEHEIGHT/CYLINDER DIAMETER

      OTHER PARAMETERS OF THE PROGRAM ARE

            H=WAVEHEIGHT/DEPTH

            TI=TIME/PERIOD

            X=HORIZONTAL DISTANCE/WAVELENGTH

            D=2*PI*DEPTH/WAVELENGTH

            BETA=D/DN=2*PI*CYLINDER RADIUS/WAVELENGTH

            CL=LIFT COEFFICIENT

            CM=ADDED MASS COEFFICIENT

            G=GRAVITATIONAL CONSTANT
```

```
      IMPLICIT REAL*8 (A-H,L-Z)
      REAL*4 DUM1,DUM2,DUM3,DUM4
      DIMENSION Y(5),U(5),S(100),VEL(100),PR(100),DE(100),DUM1(100),
     ADUM2(100,1),UDOT(100),UDOTA(100),FHN(100),FVN(100),DUM3(100,2),
     BDUM4(100,1)
    1 FORMAT (3(F20.10))
      CM=2.29
      CL=4.48
      G=32.2
      PI=3.141592653
      PI2=2.0*PI
      DO 9000 JJJ=1,15
C     READ IN VALUES
      READ (5,1) TN,HN,DN
      H=2.0*HN/DN
C     ADJUST TN FOR STAKES5
      T=TN/(PI2**2)
      WRITE (6,11) TN
   11 FORMAT ('0','TN=',F16.8)
      CALL STAKS5 (H,D,Y,U,C,CS,LO,T)
      DX=D/PI2
      WRITE (6,12)
   12 FORMAT ('0',6X,'D',11X,'Y(1)',11X,'Y(2)',11X,'Y(3)',11X,'Y(4)',
     A11X,'Y(5)',11X,'U(1)',11X,'U(2)')
      WRITE (6,14) DX,Y(1),Y(2),Y(3),Y(4),Y(5),U(1),U(2)
   14 FORMAT ('0',3X,F6.3,1P7D14.4)
      WRITE (6,13)
   13 FORMAT ('0',6X,'D',11X,'U(3)',11X,'U(4)',11X,'U(5)',11X,'C',11X,
     A'LO',11X,'CS',11X,'H')
      WRITE (6,14) DX,U(3),U(4),U(5),C,LO,CS,H
C     COMPUTE WAVE PROFILE
      BETA=D/DN
      WRITE (6,298) BETA
  298 FORMAT ('0','BETA=',1PG14.6)
C     INITIALIZE VALUES
      X=0.0
      TI=0.0
C     K DETERMINES THE NUMBER OF DATA POINTS
```

```
      K=31
      J=K-1
      DO 200 I=1,K
      TH1=PI2*(X+TI)
C     COMPUTE PHASE ANGLE
      DE(I)=(180.0/PI)*TH1
C     SET UP FOR UTPLOT
      DUM1(I)=DE(I)
      TH2=2.0*TH1
      TH3=3.0*TH1
      TH4=4.0*TH1
      TH5=5.0*TH1
C     COMPUTE WAVEPROFILE
      PR(I)=(1.0/D)*(Y(1)*DCOS(TH1)+Y(2)*DCOS(TH2)+Y(3)*DCOS(TH3)+
     AY(4)*DCOS(TH4)+Y(5)*DCOS(TH5))
C     SET UP FOR OSPLOT
      DUM2(I,1)=PR(I)
C     STEP TIME
      TI=TI+1.0/J
  200 CONTINUE
C     PLOT OF VALUES
      CALL OSPLOT (DUM1,DUM2,K,K,1,0,0.,0.,0.,0.)
C     LABEL PLOT
      WRITE (6,500)
  500 FORMAT ('0',20X,'WAVE PROFILE')
      WRITE (6,501)
  501 FORMAT ('0',20X,'X - PHASE ANGLE,DEG;Y - NONDIMENSIONAL WAVEHEIGHT
     A,Y/D')
      WRITE (6,502) TN,DN,HN,D
  502 FORMAT ('0','TN=',1PG14.4,'DN=',G14.4,'HN=',G14.4,'D=',G14.4)
C     TABLE OF VALUES
      WRITE (6,201)
```

```
  201 FORMAT ('1',10X,'PHASE ANGLE',10X,'Y VALUE')                    00000710
      WRITE (6,202) (DE(I),PR(I),I=1,K)                                 00000720
  202 FORMAT ('0',8X,1PD14.4,6X,D14.4)                                  00000730
C      DETERMINE VELOCITY VS. DEPTH                                     00000740
      ETA=PR(1)*D                                                       00000750
      STARG=D+ETA                                                       00000760
      WRITE (6,299) ETA                                                 00000770
  299 FORMAT ('0','ETA=',F16.8)                                         00000780
C     REINITIALIZE X AND TI                                             00000790
      X=0.0                                                             00000800
      TI=0.0                                                            00000810
      TH1=PI2*(X+TI)                                                    00000820
      TH2=2.0*TH1                                                       00000830
      TH3=3.0*TH1                                                       00000840
      TH4=4.0*TH1                                                       00000850
      TH5=5.0*TH1                                                       00000860
C     J DETERMINES THE NUMBER OF DATA POINTS                            00000870
      J=30                                                              00000880
C     Z= NON-DIMENSIONAL DISTANCE FROM BOTTOM=S/D                       00000890
      Z=0.0                                                             00000900
      DO 400 I=1,100                                                    00000910
      K=I                                                               00000920
      S(I)=Z                                                            00000930
C     SET UP FOR OSPLOT                                                 00000940
      DUM2(I,1)=S(I)                                                    00000950
      ARG1=D*Z                                                          00000960
      ARG2=2.0*ARG1                                                     00000970
      ARG3=3.0*ARG1                                                     00000980
      ARG4=4.0*ARG1                                                     00000990
      ARG5=5.0*ARG1                                                     00001000
      QUIT=STARG-ARG1                                                   00001010
      IF(QUIT-0.0) 401,300,300                                          00001020
C     COMPUTE VELOCITY                                                  00001030
  300 VEL(I)=C*(U(1)*DCOSH(ARG1)*DCOS(TH1)+U(2)*DCOSH(ARG2)*DCOS(TH2)+  00001040
     AU(3)*DCOSH(ARG3)*DCOS(TH3)+U(4)*DCOSH(ARG4)*DCOS(TH4)+U(5)*      00001050
     BDCOSH(ARG5)*DCOS(TH5))                                            00001060
```

```
C       SET UP FOR OSPLOT
        DUM1(I)=VEL(I)
C       STEP DEPTH
        Z=Z+1.0/J
  400   CONTINUE
C       PLOT OF VALUES
  401   DUM1(K)=0.0
        VEL(K)=0.0
        CALL OSPLOT (DUM1,DUM2,K,K,1,0,0.,0.,0.,0.)
C       LABEL PLOT
        WRITE (6,600)
  600   FORMAT ('0','HORIZONTAL VELOCITY DISTRIBUTION ON VERTICAL SECTION
       A THROUGH WAVE CREST')
        WRITE (6,601)
  601   FORMAT ('0','Y - WATER DEPTH,    , X - HORIZONTAL PARTICLE VELOCITY
       A,FT./SEC.')
        WRITE (6,603) TN,DN,HN,D
  603   FORMAT ('0','TN=', 1PG14.4,'DN=',G14.4,'HN=',G14.4,'D=',G14.4)
C       TABLE OF VELOCITY VALUES
        WRITE (6,402)
  402   FORMAT ('1',10X,'DISTANCE FROM BOTTOM',10X,'U')
        WRITE (6,403) (S(I),VEL(I),I=1,K)
  403   FORMAT ('0',13X,1PD14.4,6X,D14.4)
C       COMPUTE FORCES ON CYLINDER
C       REINITIALIZE PARAMETERS
        X=0.0
        TI=0.0
C       I DETERMINES THE NUMBER OF ITERATIONS ON PHASE ANGLE
        I=31
        III=I-1
        DO 3000 II=1,I
        RAD=(-1.0*PI)/2.0
C       DETERMINE FUNCTION ARGUMENTS
```

```
      ARG1=BETA*(1.0+DSIN(RAD))
      ARG2=ARG1*2.0
      ARG3=ARG1*3.0
      ARG4=ARG1*4.0
      ARG5=ARG1*5.0
      TH1=BETA*DCOS(RAD)+PI2*TI
      TH2=TH1*2.0
      TH3=TH1*3.0
      TH4=TH1*4.0
      TH5=TH1*5.0
      UVEL=C*(U(1)*DCOSH(ARG1)*DCOS(TH1)+U(2)*DCOSH(ARG2)*DCOS(TH2)+
     AU(3)*DCOSH(ARG3)*DCOS(TH3)+U(4)*DCOSH(ARG4)*DCOS(TH4)+U(5)*
     BDCOSH(ARG5)*DCOS(TH5))
      UVELS=UVEL**2
C     UDT= NONDIMENSIONAL X ACCELERATION
 1100 UDT=(U(1)*DCOSH(ARG1)*DSIN(TH1)+U(2)*DCOSH(ARG2)*DSIN(TH2)*2.+
     AU(3)*DCOSH(ARG3)*DSIN(TH3)*3.+U(4)*DCOSH(ARG4)*DSIN(TH4)*4.+U(5)*
     BDCOSH(ARG5)*DSIN(TH5)*5.)*D*CS
C     FV=NONDIMENSIONAL INERTAIL FORCE
      FV=DN*UVELS*CL
      DUM3(II,1)=FV
C     FHN=NONDIMENSIONAL HORIZONTAL FORCE
      FHN(II)=PI*(1.0+CM)*UDT
      DUM2(II,1)=FHN(II)
      DE(II)=(180.0/PI)*TH1
      DUM1(II)=DE(II)
C     K DETERMINES THE NUMBER OF POINTS ALONG THE CYLINDER
      K=30
C     INITIALIZE VALUES
C     PN=NONDIMENSIONAL PRESSURE BASED ON BERNOULLI EQUATION
      PN=0.0
      RAD=RAD+PI/K
      DO 2000 KK=1,K
C     SET UP FUNCTION ARGUMENTS
      ARG1=BETA*(1.0+DSIN(RAD))+PI/K
```

```
      ARG2=2.0*ARG1
      ARG3=3.0*ARG1
      ARG4=4.0*ARG1
      ARG5=5.0*ARG1
      TH1=BETA*DCOS(RAD)+PI2*TI+PI/K
      TH2=2.0*TH1
      TH3=3.0*TH1
      TH4=4.0*TH1
      TH5=5.0*TH1
C     POTS= TIME DERIVATIVE OF POTENTIAL FUNCTION NON-DIMENSIONALIZED
      POTS=CS*(U(1)*DCOSH(ARG1)*DCOS(TH1)+U(2)*DCOSH(ARG2)*DCOS(TH2)+
     AU(3)*DCOSH(ARG3)*DCOS(TH3)+U(4)*DCOSH(ARG4)*DCOS(TH4)+U(5)*
     BDCOSH(ARG5)*DCOS(TH5))
C     UVEL=NONDIMENSIONAL X VELOCITY
      UVEL=C*(U(1)*DCOSH(ARG1)*DCOS(TH1)+U(2)*DCOSH(ARG2)*DCOS(TH2)+
     AU(3)*DCOSH(ARG3)*DCOS(TH3)+U(4)*DCOSH(ARG4)*DCOS(TH4)+U(5)*
     BDCOSH(ARG5)*DCOS(TH5))
C     VVEL=NONDIMENSIONAL Y VELOCITY
      VVEL=C*(U(1)*DSINH(ARG1)*DCOS(TH1)+U(2)*DSINH(ARG2)*DCOS(TH2)+U(3)
     A*DSINH(ARG3)*DCOS(TH3)+U(4)*DSINH(ARG4)*DCOS(TH4)+U(5)*DSINH(ARG5)
     B*DCOS(TH5))
      UVELS=UVEL**2
      VVELS=VVEL**2
      PN=PN-(0.5*(UVELS+VVELS)+POTS)*DSIN(RAD)*DN*PI2/K
C     ITERATE AROUND CYLINDER SURFACE
      RAD=RAD+PI2/K
 2000 CONTINUE
      DUM3(II,2)=PN
C     COMPUTE TOTAL VERTICAL FORCE
      FVN(II)=FV+PN
      DUM4(II,1)=FVN(II)
C     ITERATE THROUGH 360 DEGREES
      TI=TI+1.0/III
 3000 CONTINUE
      CALL OSPLOT (DUM1,DUM2,I,I,1,0,0.,0.,0.,0.)
      WRITE (6,3004)
 3004 FORMAT ('0','HORIZONTAL FORCE VS. PHASE ANGLE')
```

```
      WRITE (6,3005)
 3005 FORMAT ('0',15X,'FHN',15X'PHASE ANGLE')
      WRITE (6,3006) (FHN(II),DE(II),II=1,I)
 3006 FORMAT ('0',10X,1PG14.6,10X,G14.6)
      WRITE (6,3008)
 3008 FORMAT ('1','TABLE OF VALUES FOR FV AND PN VS. PHASE ANGLE')
      WRITE (6,3009)
 3009 FORMAT ('1',10X,'LIFT COMPONENT',10X,'INERTIA  COMPONENT',10X,'PHA
     BSE ANGLE')
      WRITE (6,3010) (DUM3(II,1),DUM3(II,2),DE(II),II=1,I)
 3010 FORMAT ('0',10X,1PG14.6,11X,G14.6,10X,G14.6)
      CALL OSPLOT (DUM1,DUM4,I,I,1,0,0.,0.,0.,0.)
      WRITE (6,3011)
 3011 FORMAT ('0','TOTAL VERTICAL FORCE VS. PHASE ANGLE')
      WRITE (6,3012)
 3012 FORMAT ('0',20X,'FVN',10X,'PHASE ANGLE')
      WRITE (6,3013) (FVN(II),DE(II),II=1,I)
 3013 FORMAT ('0',13X,1PG14.6,3X,G14.6)
 9000 CONTINUE
      STOP
      END
C        SUBROUTINE STAKS5


      SUBROUTINE STAKS5 (H,D,Y,U,C,CS,LO,T)
C        DETERMINES COEFFICIENTS FOR FIFTH ORDER STOKES GRAVITY WAVE
C        INPUTS ARE: H/D(=H) AND T=G*T**2/D
      IMPLICIT REAL*8 (A-H,L-Z)
      DIMENSION Y(5,1),U(5,1)
      PI=3.141592653
      PI2=2.0*PI
C        ITERATE FOR D AND A
      D=(1.0)/T
      DO 2 J=1,100
C       NEWTON RAPHSON ITERATION ON D
      RT=(1.0/T )-D*DTANH(D)
      RTA=DABS(RT)
      HT=-1.0*(DTANH(D)+D/(DCOSH(D)**2))
```

```
C     TEST FOR CONVERGENCE
      IF (RTA-0.01) 3,1,1
C     CORRECTION
    1 D=D-(RT/HT)
    2 CONTINUE
C     VERIFY CONVERGENCE
      WRITE (6,4) RTA
    4 FORMAT ('0','D DID NOT CONVERGE IN 500 ITERATIONS,ERROR IS',F16.8)
    3 WRITE (6,5) D
    5 FORMAT ('0','D=',F16.8)
C      COMPUTE COEFFICIENTS
      DO 60 J=1,500
C     SET UP EQUATION ARGUMENTS
      C01=DCOSH(D)
      C02=C01**2
      C04=C01**4
      C06=C01**6
      C08=C01**8
      C010=C01**10
      C012=C01**12
      C014=C01**14
      C016=C01**16
      S01=DSINH(D)
      S03=S01**3
      S04=S01**4
      S05=S01**5
      S06=S01**6
      S07=S01**7
      S09=S01**9
      S010=S01**10
      S011=S01**11
      S012=S01**12
      S013=S01**13
C     COMPUTE COEFFICIENTS
      B33=3.0*(8.0*C06+1.0)/(64.0*S06)
      B35=(88128.0*C014-208224.0*C012+70848.0*C010+54000.0*C08-21816.0*
     2C06+6264.0*C04-54.0*C02-81.0)/(12288.0*S012*(6.0*C02-1.0))
```

```
      B55=(192000.0*C016-262720.0*C014+83680.0*C012+20160.0*C010-7280.0*
     2C08+7160.0*C06-1800.0*C04-1050.0*C02+225.0)/(12288.0*S010*(6.0*C02
     3-1.0)*(8.0*C04-11.0*C02+3.0))
      C1=(8.0*C04-8.0*C02+9.0)/(8.0*S04)
      C2=(3840.0*C012-4096.0*C010+2592.0*C08-1008.0*C06+5944.0*C04-1830.
     1*C02+147.0)/(512.0*S010*(6.0*C02-1.0))
C      COMPUTE A
      A=H*D/(2.0)
      DO 50 K=1,500
C     NEWTON RAPHSON ITERATION ON A
      RA=H  -(2.0*(A+(A**3)*B33+(A**5)*(B35+B55)))/D
      R=DABS(RA)
      HP=(-2.0/D)*(1.0+3.0*(A**2)*B33+5.0*(A**4)*(B35+B55))
C     TEST FOR CONVERGENCE
      IF(R-0.001) 53,49,49
C     CORRECTION
   49 A=A-(RA/HP)
   50 CONTINUE
C     VERIFY CONVERGENCE
      WRITE (6,51) RA
   51 FORMAT ('0','A DID NOT CONVERGE IN 100 ITERATIONS, ABSOLUTE ERROR
     AIS',F16.8)
   53 CO=DTANH(D)
      A2=A**2
      A4=A**4
C     ITERATE FOR PERIOD
      TE=1.0/(D*CO*(1.0+A2*C1+A4*C2))
      ER=TE-T
      ERA=DABS(ER)
      ERA=DABS(ER/T)
C     TEST FOR CONVERGENCE
      IF (ERA-0.010) 65,59,59
C     CORRECTION
```

```
   59 D=D+0.100*ER
   60 CONTINUE
      WRITE (6,61) ER
C       VERIFY CONVERGENCE
   61 FORMAT ('0','D DID NOT CONVERGE,ERROR IS',F16.8)
   65 DX=D/PI2
      WRITE (6,66) DX
   66 FORMAT ('0','D=',F16.8)
      WRITE (6,54) A
   54 FORMAT ('0','A=',F16.8)
C         DETERMINE B COEFFICIENTS
      B22=C01*(2.0*C02+1.0)/(4.0*S03)
      B24=C01*(272.0*C08-504.0*C06-192.0*C04+322.0*C02+21.0)/(384.0*S09)
      B44=C01*(768.0*C010-448.0*C08-48.0*C06+48.0*C04+106.0*C02-21.0)/
     1(384.0*S09*(6.0*C02-1.0))
C        DETERMINE WAVE CELERITY,C
      C0=DTANH(D)
      C3=(-1.0)/(4.0*S01*C01)
      C4=(12.0*C08+36.0*C06-162.0*C04+141.0*C02-27.0)/(192.0*C01*S09)
C        NOW DETERMINE WAVE EQUATION COEFFICIENTS
      A11=1.0/S01
      A13=(-1.0)*C02*(5.0*C02+1.0)/(8.0*S05)
      A15=(-1.0)*(1184.0*C010-1440.0*C08-1992.0*C06+2641.0*C04-249.0*C02
     1+18.0)/(1536.0*S011)
      A22=3.0/(8.0*S04)
      A24=(192.0*C08-424.0*C06-312.0*C04+480.0*C02-17.0)/(768.0*S010)
      A33=(13.0-4.0*C02)/(64.0*S07)
      A35=(512.0*C012+4224.0*C010-6800.0*C08-12808.0*C06+16704.0*C04-
     13154.0*C02+107.0)/(4096.0*S013*(6.0*C02-1.0))
      A44=(80.0*C06-816.0*C04+1338.0*C02-197.0)/(1536.0*S010*(6.0*C02-
     11.0))
      A55=(-1.0)*(2880.0*C010-72480.0*C08+324000.0*C06-432000.0*C04+
     1163470.0*C02-16245.0)/(61440.0*S011*(6.0*C02-1.0)*(8.0*C04-11.0*
     2C02+3.0))
C         FIND VELOCITY  AND WAVE PROFILE COEFFICIENTS AND WAVELENGTH
C            LO=WAVELENGTH
```

```
      A3=A**3
      A5=A**5
      CS=(CO/D)*(1.0+A2*C1+A4*C2)
      C=DSQRT(CS)
      Y(1,1)=A
      Y(2,1)=A2*B22+A4*B24
      Y(3,1)=A3*B33+A5*B35
      Y(4,1)=A4*B44
      Y(5,1)=A5*B55
      U(1,1)=A*A11+A3*A13+A5*A15
      U(2,1)=2.0*(A2*A22+A4*A24)
      U(3,1)=3.0*(A3*A33+A5*A35)
      U(4,1)=4.0*(A4*A44)
      U(5,1)=5.0*(A5*A55)
      LO=TE
      WRITE (6,20)
C     PRINT OUT COEFFICIENTS
   20 FORMAT ('0',6X,'D',11X,'B22',11X,'B24',11X,'B33',11X,'B35',11X,
     A'B44',11X,'B55',11X,'A11')
      WRITE (6,6) DX,B22,B24,B33,B35,B44,B55,A11
    6 FORMAT ('0',3X,F6.3,1P7D14.4)
      WRITE (6,7)
    7 FORMAT ('0',6X,'D',11X,'A13',11X,'A15',11X,'A22',11X,'A24',11X,
     A'A33',11X,'A35',11X,'A44')
      WRITE (6,6) DX,A13,A15,A22,A24,A33,A35,A44
      WRITE (6,8)
    8 FORMAT ('0',6X,'D',11X,'A55',11X,'C1',11X,'C2',11X,'C3',11X,'C4',
     A11X,'SO1',11X,'CO1')
      WRITE (6,6) DX,A55,C1,C2,C3,C4,SO1,CO1
      RETURN
      END
```

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ABSTRACT

A horizontal cylinder, located near the floor of a two dimensional wave channel was subjected to a train of non-linear gravity waves. The horizontal and vertical forces were measured and presented in dimensionless form. Experimental values of horizontal and vertical force coefficients are presented as functions of dimensionless wave height and dimensionless wave period. The dimensionless force coefficients predicted by a modified Morrison equation and a Froude-Krilov force are compared to experimental data. Fluid particle velocity and acceleration values were calculated from Stokes fifth-order gravity wave theory. Experimental dimensionless wave periods from 10 to 200 were investigated.

The horizontal force coefficients were found to vary linearly with wave height for dimensionless period values from 60 to 120. The vertical force coefficients were found to be inertia dominated at low dimensionless wave heights and dominated by a lift force at higher wave heights. The theory predicted experimental values of force coefficients with good accuracy, especially at greater water depths.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
gravity wave forces						
non-linear						
wave interaction with underwater cylindrical bodies						

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